PRIIMEK	IME	VPISNA ŠTEVILKA	SMER

NALOGA	TOČKE
1.	
2.	
3.	
SKUPAJ	

MATEMATIČNA ANALIZA 3

1. kolokvij - računski del

7.12.2004

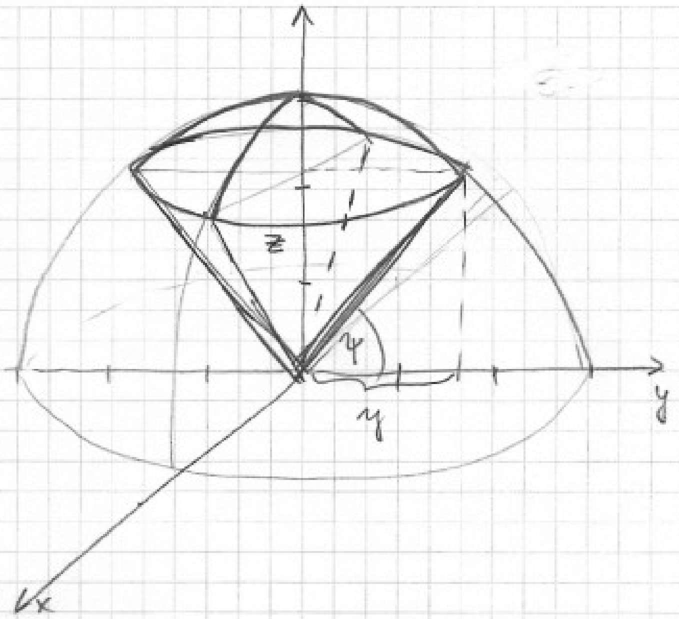
Točkovanje: 35+30+35=100

1. Narišite skico telesa

$$G = \{(x,y,z) \in \mathbb{R}^3;\ \sqrt{3(x^2+y^2)} \le z \le \sqrt{9-x^2-y^2}\}$$

in izračunajte njegov volumen.

Namig : Uporabite sferične koordinate.



$$z^2 = 9 - x^2 - y^2$$
$$x^2 + y^2 + z^2 = 9$$
$$z = \sqrt{3(x^2+y^2)}$$
$$z^2 = 3\,(x^2+y^2)$$

PRESEK

$$\left.\begin{array}{l} z^2 = 9 - x^2 - y^2 \\ z^2 = 3(x^2+y^2) \end{array}\right\} \Rightarrow$$

$$9 - (x^2+y^2) = 3(x^2+y^2)$$
$$9 = 4(x^2+y^2)$$
$$\frac{9}{4} = x^2+y^2 \Rightarrow$$
$$r = \frac{3}{2}$$

$x = 0$: $z = \sqrt{3}\,y$

$\Rightarrow \operatorname{tg}\psi = \sqrt{3} \to \psi = \frac{\pi}{3}$

SFERIČNE KOORDINATE

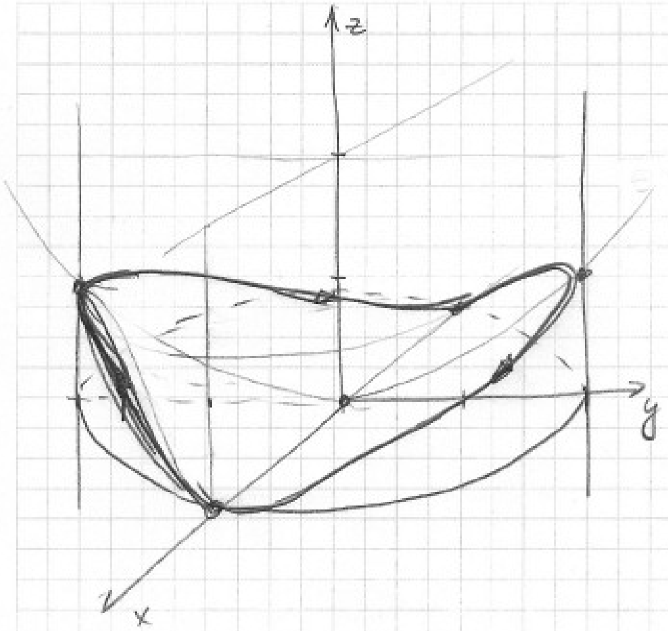
$$x = r\cos\varphi\cos\psi$$
$$y = r\sin\varphi\cos\psi$$
$$z = r\sin\psi$$
$$J = r^2\cos\psi$$

$$V = \int_0^{2\pi} d\varphi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} d\psi \int_0^3 r^2\cos\psi\, dr$$

$$= 2\pi\left[\sin\psi\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left[\frac{r^3}{3}\right]_0^3$$

$$= 2\pi\left(1 - \frac{\sqrt{3}}{2}\right)\frac{27}{3} = 9\pi(2-\sqrt{3})$$

2. Zapišite enačbo krivulje $\vec{K}$, ki je presek valja $x^2+y^2=1$ in ploskve $z=\frac{y^2}{2}$ ter je orientirana v smer urinega kazalca, če jo gledamo it $T(0,0,10)$. Skicirajte krivuljo $\vec{K}$. Izračunajte delo, ki ga opravi sila $\vec{F}(x,y,z)=(-y,z,0)$, ko premakne masni delec vzdolž krivulje $\vec{K}$.



$z=\frac{y^2}{2}$

x	y	z
1	0	0
0	1	$\frac{1}{2}$
-1	0	0
0	-1	$\frac{1}{2}$

$$\vec{p}(\varphi)=\left(\cos\varphi,\ \sin\varphi,\ \frac{\sin^2\varphi}{2}\right)$$

$$\dot{\vec{p}}(\varphi)=\left(-\sin\varphi,\ \cos\varphi,\ \frac{\cancel{2}\sin\varphi\cos\varphi}{\cancel{2}}\right)=\left(-\sin\varphi,\ \cos\varphi,\ \sin\varphi\cos\varphi\right)$$

$$(\vec{F}\circ\vec{p})(\varphi)=\vec{F}\left(\cos\varphi,\ \sin\varphi,\ \frac{\sin^2\varphi}{2}\right)=\left(-\sin\varphi,\ \frac{\sin^2\varphi}{2},\ 0\right)$$

$$A=\int_{\vec{K}}\vec{F}\,d\vec{r}=-\int_0^{2\pi}(\vec{F}\circ\vec{p})(\varphi)\,\dot{\vec{p}}(\varphi)\,d\varphi$$

$$=-\int_0^{2\pi}\left(-\sin\varphi,\ \frac{\sin^2\varphi}{2},\ 0\right)\left(-\sin\varphi,\ \cos\varphi,\ \sin\varphi\cos\varphi\right)d\varphi$$

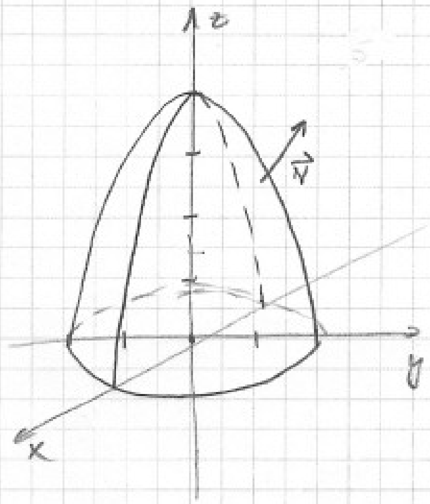
$$=-\int_0^{2\pi}\left(\sin^2\varphi+\frac{\sin^2\varphi\cos\varphi}{2}+0\right)d\varphi$$

$$=-\int_0^{2\pi}\sin^2\varphi\,d\varphi-\underbrace{\frac{1}{2}\int_0^{2\pi}\sin^2\varphi\cos\varphi\,d\varphi}_{0}=-\cancel{4}^{2}\,\frac{1}{\cancel{2}}B\left(\frac{3}{2},\frac{1}{2}\right)$$

$$=-\frac{2\cdot\frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{1}=\boxed{-\pi}$$

3. Izračunajte pretok vektorskega polja $\vec{a}\,(x,y,z) = (z, xy, 1)$ skozi zgornjo stran paraboloida

$$P = \{(x,y,z) \in \mathbb{R}^3; z = 4 - x^2 - y^2, 0 \leq z \leq 4\}$$



$z=0 \;:\; 4-x^2-y^2=0$

$x^2+y^2=4$

$z=4 \quad 4-x^2-y^2=4$

$x^2+y^2=0$

$x=0 \;:\; z=4-y^2$

$y=0 \quad z=4-x^2$

$\vec p(r,\varphi) = (r\cos\varphi, r\sin\varphi, 4-r^2) \qquad \varphi \in [0,2\pi]$

$\vec p_\varphi = (-r\sin\varphi, r\cos\varphi, 0) \qquad r \in [0,2]$

$\vec p_r = (\cos\varphi, \sin\varphi, -2r)$

$$\Rightarrow \vec p_\varphi \times \vec p_r = \begin{vmatrix} \vec i & \vec j & \vec k \\ -r\sin\varphi & r\cos\varphi & 0 \\ \cos\varphi & \sin\varphi & -2r \end{vmatrix} = (-2r^2\cos\varphi, -2r\sin^2\varphi, \overset{<0}{-r})$$

$\Rightarrow$ ORIENTACIJI

$\vec p_\varphi \times \vec p_r$ in $\vec N$

STA NASPROTNI

$$\iint_{\vec P} \vec a\, \vec{dS} = -\iint_U (4-r^2, r^2\cos\varphi\sin\varphi, 1)(-2r^2\cos\varphi, -2r\sin^2\varphi, -r)\,dr\,d\varphi$$

$$= \iint_U (\cancel{8r^2\cos\varphi} - \cancel{2r^4\cos\varphi} + \cancel{2r^3\cos\varphi\sin^2\varphi} + r)\,dr\,d\varphi$$

$$= \iint_U r\,dr\,d\varphi = \int_0^{2\pi} d\varphi \int_0^2 r\,dr = 2\pi\left[\frac{r^2}{2}\right]_0^2 = \boxed{4\pi}$$

Ausplicchus

$$z = 4 - x^2 - y^2$$

$$D = \{(x,y) \in \mathbb{R}^2 \; ; \; x^2 + y^2 \leq 4\}$$

$$p = z_x = -2x$$

$$q = z_y = -2y$$

$$\iint_{\vec{P}} \vec{F}\, \vec{dS} = + \iint_D (4 - x^2 - y^2, xy, 1)(2x, 2y, 1)\, dx\, dy$$

$$= \iint_D (8x - 2x^3 \cancel{- 2xy^2} + \cancel{2xy^2} + 1)\, dx\, dy$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$J = r$

$$= \int_0^{2\pi} d\varphi \int_0^2 (8r\cos\varphi - 2r^3\cos^3\varphi + 1)\, r\, dr \quad \sin^2\varphi +$$

$$= \cancel{8\int_0^{2\pi} \cos\varphi\, d\varphi} \int_0^2 r^2\, dr - 2 \cancel{\int_0^{2\pi} \cos^3\varphi\, d\varphi} \int_0^2 r^4\, dr + \int_0^{2\pi} d\varphi \int_0^2 r\, dr$$

$$= 2\pi \left[\frac{r^2}{2}\right]_0^2 = \boxed{4\pi}$$