DN.

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$$\vec{r}(t) = \left(t, \operatorname{arcsin} t, \frac{1}{4}\ln\frac{1+t}{1-t}\right), \quad t \in [0, \tfrac{1}{2}]$$

ALI JE $\vec{r}(t)$ ENOSTAVNA IN REGULARNA?

(a) $\vec{r}(t_1) = \vec{r}(t_2) \Rightarrow$

$$\left(t_1, \operatorname{arcsin} t_1, \frac{1}{4}\ln\frac{1+t_1}{1-t_1}\right) = \left(t_2, \operatorname{arcsin} t_2, \frac{1}{4}\ln\frac{1+t_2}{1-t_2}\right) \Rightarrow$$

$\Rightarrow$ ① $t_1 = t_2$

② $\operatorname{arcsin} t_1 = \operatorname{arcsin} t_2$

③ $\frac{1}{4}\ln\frac{1+t_1}{1-t_1} = \frac{1}{4}\ln\frac{1+t_2}{1-t_2}$

$\Rightarrow t_1 = t_2 \Rightarrow \vec{r}(t)$ JE ENOSTAVNA

(b) $\dot{\vec{r}}(t) = \left(1, \frac{1}{\sqrt{1-t^2}}, \frac{1}{2(1-t^2)}\right) \Rightarrow \vec{r}(t)$ JE RAZREDA C^1

$$\left(\ln\frac{1+t}{1-t}\right)' = \frac{1-t}{1+t}\cdot\frac{1-t+1+t}{(1-t)^2} = \frac{2}{1-t^2} \quad (*)$$

KER JE TUDI $\dot{\vec{r}}(t) \neq \vec{0}$ ZA VSAK $t \in [0, \frac{1}{2}]$, JE $\vec{r}(t)$ REGULARNA.

$$\|\dot{\vec{r}}(t)\|^2 = 1^2 + \frac{1}{1-t^2} + \frac{1}{4(1-t^2)^2} = \frac{4(1-t^2)^2 + 4(1-t^2) + 1}{4(1-t^2)^2}$$

$$= \frac{(2(1-t^2)+1)^2}{4(1-t^2)} = \left(\frac{2(1-t^2)+1}{2(1-t^2)}\right)^2 = \left(1 + \frac{1}{2(1-t^2)}\right)^2$$

$$s(K) = s(\vec{r}) = \int_0^{\frac{1}{2}} \|\dot{\vec{r}}(t)\|\,dt = \int_0^{\frac{1}{2}} \left(1 + \frac{1}{2(1-t^2)}\right) dt$$

$$= t\Big|_0^{\frac{1}{2}} + \frac{1}{4}\int_0^{\frac{1}{2}} \frac{2}{1-t^2}\,dt \overset{(*)}{=} \frac{1}{2} + \frac{1}{4}\ln\frac{1+t}{1-t}\Big|_0^{\frac{1}{2}}$$

$$= \frac{1}{2} + \frac{1}{4}\left(\ln\frac{\frac{3}{2}}{\frac{1}{2}\cdot 1} - \ln\frac{1}{1}\right) = \frac{1}{2} + \frac{1}{4}\ln 3$$