MSK

NAJ BO $\vec{p}(t) = \left(\frac{1}{2}\sin^2 t, \frac{1}{2}(t + \sin t \cos t), \sin t\right), t \in [0, \frac{\pi}{4}]$

IZRAČUNAJ DOLŽINO POTI $\vec{p}(t)$.

ALI JE $\vec{p}(t)$ ENOSTAVNA IN REGULARNA

(a) $\vec{p}(t_1) = \vec{p}(t_2)$

$\left(\frac{1}{2}\sin^2 t_1, \frac{1}{2}(t_1 + \sin t_1 \cos t_1), \sin t_1\right) = \left(\frac{1}{2}\sin^2 t_2, \frac{1}{2}(t_2 + \sin t_2 \cos t_2), \sin t_2\right)$

$$\Rightarrow \left.\begin{array}{l} \sin^2 t_1 = \sin^2 t_2 \\ t_1 + \sin t_1 \cos t_1 = t_2 + \sin t_1 \cos t_1 \\ \sin t_1 = \sin t_2 \end{array}\right\} \Rightarrow t_1 = t_2$$

, KER SKLO NA INTERVALU $[0, \frac{\pi}{4}]$

(b) $\dot{\vec{p}}(t) = \left(\sin t \cos t, \frac{1}{2} + \frac{1}{2}\cos^2 t - \frac{1}{2}\sin^2 t, \cos t\right)$

$= (\sin t \cos t, \cos^2 t, \cos t)$

$$\|\dot{\vec{p}}(t)\|^2 = \sin^2 t \cos^2 t + \cos^4 t + \cos^2 t$$

$$= \cos^2 t\,(\sin^2 t + \cos^2 t + 1)$$

$$= 2\cos^2 t \neq 0 \quad \text{ZA } t \in [0, \tfrac{\pi}{4}]$$

$$s(\vec{p}) = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 t}\, dt = \sqrt{2}\int_0^{\frac{\pi}{4}} |\cos t|\, dt = \sqrt{2}\int_0^{\frac{\pi}{4}} \cos t\, dt$$

$$= \sqrt{2}\sin t\Big|_0^{\frac{\pi}{4}} = \sqrt{2}\left(\sin\frac{\pi}{4} - \sin 0\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = \underline{1}$$