DN1

DN

IZRAČUNAJ DOLŽINO KRIVULJE $2y = x^2$, $3z = xy$ OD TOČKE $T_1(0,0,0)$ DO TOČKE $T_2(2,2,\frac{4}{3})$

UVEDEMO PARAMETER: $x = t \Rightarrow y = \frac{x^2}{2} = \frac{t^2}{2}$

$$z = \frac{xy}{3} = \frac{t\frac{t^2}{2}}{3} = \frac{t^3}{6}$$

$\vec{p}(t) = (t, \frac{t^2}{2}, \frac{t^3}{6})$ $\quad t \in [0,2]$

- $\vec{p}(0) = 0$
- $\vec{p}(2) = (2, 2, \frac{4}{3})$

Ali je $K = \vec{p}([0,2])$ enostavno sklenjena krivulja razreda C^1?

- $\dot{\vec{p}}(t) = (1, t, \frac{t^2}{2})$ JE ZVEZNA $\Rightarrow \vec{p}(t) \in C^1(\mathbb{R})$ $(\vec{p} \in C^\infty(\mathbb{R}))$
- $\dot{\vec{p}}(t) = (1, t, \frac{t^2}{2}) \neq \vec{0}$ ZA VSAK $t \in [0,2] \Rightarrow \vec{p}(t)$ sklenjena
- INJEKTIVNOST NA $[0,2]$:

$$\vec{p}(t_1) = \vec{p}(t_2) \Rightarrow (t_1, \frac{t_1^2}{2}, \frac{t_1^3}{6}) = (t_2, \frac{t_2^2}{2}, \frac{t_2^3}{6})$$

$$\Rightarrow t_1 = t_2 \Rightarrow \vec{p}(t) \text{ JE INJ. NA } [0,2]$$

$$l(K) = \int_0^2 \|\dot{\vec{p}}(t)\| \, dt = \int_0^2 \frac{2+t^2}{2} \, dt = \int_0^2 (1 + \frac{t^2}{2}) \, dt =$$

$$\|\dot{\vec{p}}(t)\|^2 = 1 + t^2 + \frac{t^4}{4} = \frac{4 + 4t^2 + t^4}{4} = \left(\frac{2+t^2}{2}\right)^2$$

$$= \left[t + \frac{t^3}{6}\right]_0^2 = 2 + \frac{8}{6} = 2 + \frac{4}{3} = \frac{10}{3}$$