DOLOČI $\vec{T}, \vec{N}, \vec{B}$, $\varkappa$ IN τ, ČE JE

$$\vec{r}(t) = (\ln(\cos t), \ln(\sin t), \sqrt{2}t) \quad ; t \in (0, \tfrac{\pi}{2})$$

$$\dot{\vec{r}}(t) = \left(\frac{-\sin t}{\cos t}, \frac{\cos t}{\sin t}, \sqrt{2}\right) = \frac{1}{\cos t \sin t}\left(-\sin^2 t, \cos^2 t, \sqrt{2}\sin t \cos t\right)$$

$$\ddot{\vec{r}}(t) = \left(\frac{-\cos^2 t - \sin^2 t}{\cos^2 t}, \frac{-\sin^2 t - \cos^2 t}{\sin^2 t}, 0\right) = \left(\frac{-1}{\cos^2 t}, \frac{-1}{\sin^2 t}, 0\right)$$

$$\dddot{\vec{r}}(t) = \left(\frac{-2\sin t}{\cos^3 t}, \frac{2\cos t}{\sin^3 t}, 0\right)$$

$$\|\dot{\vec{r}}(t)\| = \sqrt{\frac{\sin^2 t}{\cos^2 t} + \frac{\cos^2 t}{\sin^2 t} + 2} = \sqrt{\frac{\sin^4 t + \cos^4 t + 2\cos^2 t \sin^2 t}{\cos^2 t \sin^2 t}}$$

$$= \sqrt{\frac{(\sin^2 t + \cos^2 t)^2}{\cos^2 t \sin^2 t}} = \frac{1}{|\cos t \sin t|} = \frac{1}{\cos t \sin t} \qquad (\text{KER } t \in (0, \tfrac{\pi}{2}))$$

$$\vec{T} = \frac{\dot{\vec{r}}(t)}{\|\dot{\vec{r}}(t)\|} = \left(-\sin^2 t, \cos^2 t, \sqrt{2}\sin t \cos t\right)$$

$$\dot{\vec{r}}(t) \times \ddot{\vec{r}}(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{-\sin t}{\cos t} & \frac{\cos t}{\sin t} & \sqrt{2} \\ \frac{-1}{\cos^2 t} & \frac{-1}{\sin^2 t} & 0 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ \frac{-\sin t}{\cos t} & \frac{\cos t}{\sin t} \\ \frac{-1}{\cos^2 t} & \frac{-1}{\sin^2 t} \end{matrix}$$

$$= \left(\frac{\sqrt{2}}{\sin^2 t}, \frac{-\sqrt{2}}{\cos^2 t}, \frac{1}{\sin t \cos t} + \frac{1}{\sin t \cos t}\right) = \left(\frac{\sqrt{2}}{\sin^2 t}, \frac{-\sqrt{2}}{\cos^2 t}, \frac{2}{\sin t \cos t}\right)$$

$$\|\dot{\vec{r}}(t) \times \ddot{\vec{r}}(t)\| = \sqrt{\frac{2}{\sin^4 t} + \frac{2}{\cos^4 t} + \frac{4}{\sin^2 t \cos^2 t}} = \sqrt{2\,\frac{(\cos^4 t + \sin^4 t + 2\sin^2 t \cos^2 t)}{\sin^4 t \cos^4 t}}$$

$$= \sqrt{\frac{2(\cos^2 t + \sin^2 t)^2}{\sin^4 t \cos^4 t}} = \frac{\sqrt{2}}{\cos^2 t \sin^2 t}$$

$$\vec{B}(t) = \frac{\dot{\vec{r}}(t) \times \ddot{\vec{r}}(t)}{\|\dot{\vec{r}}(t) \times \ddot{\vec{r}}(t)\|} = \frac{\frac{\sqrt{2}}{\sin^2 t \cos^2 t}\left(\cos^2 t, -\sin^2 t, \sqrt{2}\sin t \cos t\right)}{\frac{\sqrt{2}}{\cos^2 t \sin^2 t}}$$

$$\Rightarrow \vec{B}(t) = \left(\cos^2 t, -\sin^2 t, \sqrt{2}\sin t \cos t\right)$$

$$\vec{N}(t) = \vec{B}(t) \times \vec{T}(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ \cos^2 t & -\sin^2 t & \sqrt{2}\sin t\cos t & \cos^2 t & -\sin^2 t \\ -\sin^2 t & \cos^2 t & \sqrt{2}\sin t\cos t & -\sin^2 t & \cos^2 t \end{vmatrix}$$

$$= \left(-\sqrt{2}\sin^3 t\cos t - \sqrt{2}\sin t\cos^3 t\ ,\ -\sqrt{2}\sin^3 t\cos t - \sqrt{2}\sin t\cos^3 t\ ,\ \cos^4 t - \sin^4 t\right)$$

$$= \left(-\sqrt{2}\sin t\cos t\ ,\ -\sqrt{2}\sin t\cos t\ ,\ (\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t)\right)$$

$$= \left(-\frac{\sqrt{2}}{2}\sin 2t\ ,\ -\frac{\sqrt{2}}{2}\sin 2t\ ,\ \cos 2t\right)$$

$$\boxed{\vec{N}(t) = \frac{\sqrt{2}}{2}\left(-\sin 2t\ ,\ -\sin 2t\ ,\ \sqrt{2}\cos 2t\right)}$$

$$\kappa(t) = \frac{\|\dot{\vec{r}}(t) \times \ddot{\vec{r}}(t)\|}{\|\dot{\vec{r}}(t)\|^3} = \frac{\sqrt{2}\ \cos^3 t \sin^3 t}{\cos^2 t \sin^2 t} = \sqrt{2}\cos t\sin t = \boxed{\frac{\sqrt{2}}{2}\sin 2t}$$

$$\left[\dot{\vec{r}}(t), \ddot{\vec{r}}(t), \dddot{\vec{r}}(t)\right] = \frac{1 \cdot (-1) \cdot 2}{\sin t\cos t\ \cos^2 t\ \sin^2 t\ \cos^3 t \sin^3 t} \begin{vmatrix} -\sin^2 t & \cos^2 t & \sqrt{2}\sin t\cos t \\ \sin^2 t & \cos^2 t & 0 \\ -\sin^4 t & \cos^4 t & 0 \end{vmatrix}$$

$$= \frac{-2\sqrt{2}\sin t\cos t}{\sin^6 t\cos^6 t}\left(\sin^2 t\cos^4 t + \sin^4 t\cos^2 t\right)$$

$$= \frac{-2\sqrt{2}\ \sin^2 t\cos^2 t}{\sin^5 t\cos^5 t}\left(\cos^2 t + \sin^2 t\right) = \boxed{\frac{-2\sqrt{2}}{\sin^3 t\cos^3 t}}$$

$$\tau(t) = \frac{\left[\dot{\vec{r}}(t), \ddot{\vec{r}}(t), \dddot{\vec{r}}(t)\right]}{\|\ddot{\vec{r}}(t) \times \dddot{\vec{r}}(t)\|^2} = \frac{-2\sqrt{2}\ \ \cos^4 t + \sin^4 t}{\sin^3 t\cos^3 t\ \ 2} = -\sqrt{2}\sin t\cos t = \boxed{-\frac{\sqrt{2}}{2}\sin 2t}$$