DN

(11) Skicaj krivuljo $x(t) = t^2$, $y(t) = t - \frac{1}{3}t^3$ in izračunaj dolžino zanke!

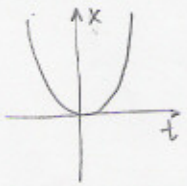
$x(t)$

$D_{x(t)} = \mathbb{R}$

$N: t = 0$

$\dot{x}(t) = 2t = 0 \Rightarrow t = 0$

$\Rightarrow x(t)$ narašča, če je $t > 0$



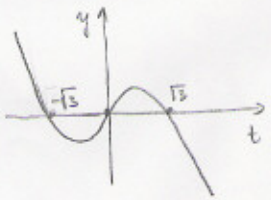
$y(t) = t - \frac{1}{3}t^3 = t\left(1 - \frac{1}{3}t^2\right)$

$D_{y(t)}: \mathbb{R}$

$N: t = 0, \; t = \pm\sqrt{3}$

$\dot{y}(t) = 1 - t^2 = 0 \to t = \pm 1$

$\Rightarrow y$ narašča, če je $t \in (-1, 1)$



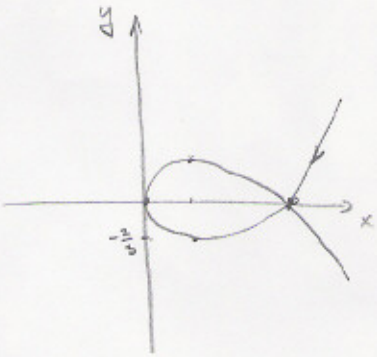
t	$-\infty$		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$		∞
x	∞	$\searrow$	3	$\searrow$	1	$\searrow$	0	$\nearrow$	1	$\nearrow$	3	$\nearrow$	∞
y	∞	$\searrow$	0	$\searrow$	$-\frac{2}{3}$	$\nearrow$	0	$\nearrow$	$\frac{2}{3}$	$\searrow$	0	$\searrow$	$-\infty$

$\dot{y}(x) = \frac{\dot{y}}{\dot{x}} = \frac{1 - t^2}{2t} = \varphi(t)$

$y''(x) = \frac{\dot{\varphi}}{\dot{x}} = \frac{-2t(2t) - (1-t^2)\cdot 2}{4t^2 \cdot 2t} =$

$= \frac{-4t^2 - 2 + 2t^2}{8t^3} = \frac{-1 - t^2}{4t^3} = \frac{-(1+t^2)}{4t^3}$

$\Rightarrow y''(x) > 0$, če je $t < 0$



$$A(K) = \int_{-\sqrt{3}}^{\sqrt{3}} \|\dot{\vec{p}}(t)\| \, dt = \int_{-\sqrt{3}}^{\sqrt{3}} |1 + t^2| \, dt = 2\int_0^{\sqrt{3}} (1 + t^2) \, dt = 2\left[t + \frac{t^3}{3}\right]_0^{\sqrt{3}} = 2\left[\sqrt{3} + \frac{3\sqrt{3}}{3}\right] = 4\sqrt{3}$$

$\dot{\vec{p}}(t) = (2t, 1 - t^2) \Rightarrow \|\dot{\vec{p}}\|^2 = 4t^2 + 1 - 2t^2 + t^4 = 1 + 2t^2 + t^4 = (1 + t^2)^2$