DN

DN IZRAČUNAJ $\int_K u\, ds$, ČE JE

FORMULA
$$\int_K u\, ds = \int_a^b u(\vec{p}(t))\, \|\dot{\vec{p}}(t)\|\, dt$$

ZA $u(x,y) = x^{\frac{4}{3}} + y^{\frac{4}{3}}$

IN $K: \quad \vec{p}(t) = (a\cos^3 t, a\sin^3 t) \quad t \in [0, \frac{\pi}{2}]$

$\dot{\vec{p}}(t) = (-3a\cos^2 t \sin t,\ 3a\sin^2 t \cos t)$

$$\|\dot{\vec{p}}(t)\|^2 = 9a^2\cos^4 t \sin^2 t + 9a^2 \sin^4 t \cos^2 t$$
$$= 9a^2\cos^2 t \sin^2 t\,(\cos^2 t + \sin^2 t)$$
$$= 9a^2 \cos^2 t \sin^2 t$$

$$\int_\gamma u\, ds = \int_0^{\frac{\pi}{2}} \underbrace{a^{4/3}(\cos^4 t + \sin^4 t)}_{u(\vec{p}(t))} \cdot \underbrace{3a\cos t \sin t}_{\|\dot{\vec{p}}(t)\|}\, dt$$
$$= 3a^{7/3} \int_0^{\pi/2} [\cos^5 t \sin t + \sin^5 t \cos t]\, dt$$
$$= 3a^{7/3}\left[\tfrac{1}{2}B(1,3) + \tfrac{1}{2}B(3,1)\right]$$
$$= 3a^{7/3}\, B(3,1) = 3a^{7/3}\,\frac{\Gamma(3)\Gamma(1)}{\Gamma(4)}$$
$$= 3a^{7/3}\,\frac{2}{3\cdot 2} = a^{7/3}$$