D.N.

D11.

DOLOČITE MASO KRIVULJE

$$\gamma: \quad \vec{p}(t) = \left(a(\cos t + t\sin t),\ a(\sin t - t\cos t),\ \tfrac{1}{2}bt^2\right), \quad t \in [0, 2\pi],$$

ČE JE $\varrho(x,y) = k(x^2+y^2)$

$$m = \int_\gamma \varrho \, ds = \int_0^{2\pi} \underbrace{a^2k(1+t^2)}_{k(x^2+y^2)} \underbrace{\sqrt{t^2(a^2+b^2)}}_{\|\dot{\vec{p}}(t)\|} dt \overset{*}{=}$$

$$x^2 + y^2 = a^2(\cos t + t\sin t)^2 + a^2(\sin t - t\cos t)^2$$

$$= a^2\left(\cos^2 t + 2t\cancel{\cos t \sin t} + t^2\sin^2 t + \sin^2 t - 2t\cancel{\sin t\cos t} + t^2\cos^2 t\right)$$

$$= a^2(1 + t^2)$$

$$\dot{\vec{p}}(t) = \left(a(\cancel{-\sin t} + \cancel{\sin t} + t\cos t),\ a(\cancel{\cos t} - \cancel{\cos t} + t\sin t),\ \tfrac{2}{\cancel{2}}bt\right)$$

$$= (at\cos t,\ at\sin t,\ bt)$$

$$\|\dot{\vec{p}}(t)\|^2 = \underbrace{a^2t^2\cos^2 t + a^2t^2\sin^2 t}_{} + b^2t^2$$

$$= a^2t^2 + b^2t^2 = t^2(a^2+b^2)$$

$$\overset{*}{=} a^2k\sqrt{a^2+b^2} \int_0^{2\pi} (1+t^2)t\,dt = a^2k\sqrt{a^2+b^2}\left[\frac{t^2}{2} + \frac{t^4}{4}\right]_0^{2\pi}$$

$$= a^2k\sqrt{a^2+b^2}\left[\frac{4\pi^2}{\cancel{2}} + \frac{16\pi^4}{\cancel{4}}\right] = \boxed{2a^2\sqrt{a^2+b^2}\,\pi^2k(1+2\pi^2)}$$