D.N

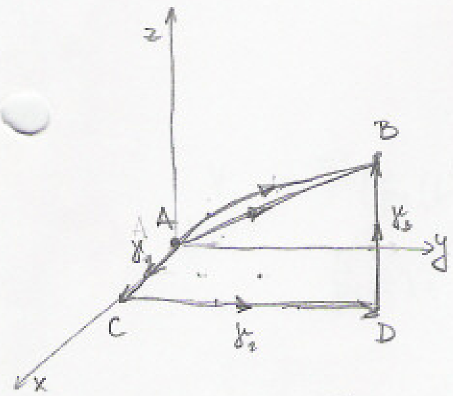
IZRAČUNAJ $\int_{\gamma, A}^{B} \vec{F}\, d\vec{r}$, ČE JE $\vec{F}(x,y,z) = (yz, xz, xy+1)$

IN

(a) $\gamma : \vec{p}(t) = (t, 3t^2, 2t)$; $t \in [0,1]$

(b) $\gamma : \vec{p}(t) = t(1,3,2)$; $t \in [0,1]$

(c) $\gamma = \gamma_1 \cup \gamma_2 \cup \gamma_3$



$\gamma_1 : \vec{p}_1(t) = (t, 0, 0)$; $t \in [0,1]$

$\gamma_2 : \vec{p}_2(t) = (1, t, 0)$; $t \in [0,3]$

$\gamma_3 : \vec{p}_3(t) = (1, 3, t)$; $t \in [0,2]$

(a) $$\int_{\gamma, A}^{B} \vec{F} = \int_0^1 \vec{F}(\vec{p}(t))\,\dot{\vec{p}}(t)\,dt = \int_0^1 (6t^3, 2t^2, 3t^3+1)(1, 6t, 2)\,dt \overset{*}{=}$$

$$\vec{F}(\vec{p}(t)) = \vec{F}(t, 3t^2, 2t) = (3t^2 \cdot 2t, t \cdot 2t, t \cdot 3t^2 + 1) =$$
$$= (6t^3, 2t^2, 3t^3+1)$$

$$\dot{\vec{p}}(t) = (1, 6t, 2)$$

$$\overset{*}{=} \int_0^1 [6t^3 + 12t^3 + 6t^3 + 2]\,dt = \int_0^1 [24t^3 + 2]\,dt = \left[\frac{\cancel{24}^{6} t^4}{\cancel{4}_{1}} + 2t\right]_0^1 = 8$$

(b) $$\int_{\gamma, A}^{B} \vec{F} = \int_0^1 \vec{F}(\vec{p}(t))\,\dot{\vec{p}}(t)\,dt = \int_0^1 (6t^2, 2t^2, 3t^2+1)(1,3,2) \overset{*}{=}$$

$$\vec{F}(\vec{p}(t)) = \vec{F}(t, 3t, 2t) = (3t \cdot 2t, t \cdot 2t, t \cdot 3t + 1) = (6t^2, 2t^2, 3t^2+1)$$

$$\dot{\vec{p}}(t) = (1,3,2)$$

$$\overset{*}{=} \int_0^1 [6t^2 + 6t^2 + 6t^2 + 2]\,dt = \int_0^1 [18t^2+2]\,dt = \left[\frac{\overset{6}{\cancel{18}}t^3}{\cancel{3}} + 2t\right]_0^1 = \boxed{8}$$

(c) $$\int_{\gamma:A}^{B} \vec{F} = \int_{\gamma_1:A}^{C} \vec{F} + \int_{\gamma_2:C}^{D} \vec{F} + \int_{\gamma_3:D}^{B} \vec{F}$$

$$= \int_0^1 \underbrace{(0,0,1)(1,0,0)}_{0}\,dt + \int_0^3 \underbrace{(0,0,t+1)(0,1,0)}_{0}\,dt + \int_0^2 (3t,t,4)(0,0,1)\,dt$$

γ_1: $\vec{F}(\vec{p}_1(t)) = \vec{F}(t,0,0) = (0,0,1)$

$\dot{\vec{p}}_1(t) = (1,0,0)$

γ_2: $\vec{F}(\vec{p}_2(t)) = \vec{F}(1,t,0) = (t\cdot 0, 1\cdot 0, 1\cdot t+1) = (0,0,t+1)$

$\dot{\vec{p}}_2(t) = (0,1,0)$

γ_3: $\vec{F}(\vec{p}_3(t)) = \vec{F}(1,3,t) = (3t, t, 3+1) = (3t,t,4)$

$\dot{\vec{p}}_3(t) = (0,0,1)$

$$\overset{*}{=} 0 + 0 + \int_0^2 4\,dt = \boxed{8}$$

OPOMBA: $\vec{F}$ JE GRADIENTNO (KONZERVATIVNO) POLJE:

$\vec{F} = \operatorname{grad} u$, ZA $u(x,y,z) = (xy+1)z$,

ZATO JE $$\int_{\gamma:A}^{B} \vec{F} = u(B) - u(A) = u(1,3,2) - u(0,0,0)$$

$$= (1\cdot 3+1)\cdot 2 - 0 = \boxed{8}$$