DN 1/6

(16) $\int_K yz\,dx + zx\,dy + xy\,dz$ gdje je K luk spirale

$x = R\cos t$, $y = R\sin t$, $z = \frac{at}{2\pi}$ od presjecišta s ravninom $z=0$

do presjecišta s ravninom $z = a$!

$$z = 0 \Rightarrow \frac{at}{2\pi} = 0 \Rightarrow t = 0$$

$$z = a \Rightarrow \frac{at}{2\pi} = a \Rightarrow t = 2\pi$$

$$x = R\cos t \Rightarrow dx = -R\sin t$$

$$y = R\sin t \Rightarrow dy = R\cos t$$

$$z = \frac{at}{2\pi} \Rightarrow dz = \frac{a}{2\pi}dt$$

$$I = \int_0^{2\pi}\left[R\sin t \cdot \frac{at}{2\pi}(-R\sin t) + \frac{at}{2\pi} R\cos t\, R\cos t + R\cos t\, R\sin t\, \frac{a}{2\pi}\right]dt$$

$$= \int_0^{2\pi}\left[-\frac{R^2a}{2\pi} t\sin^2 t + \frac{R^2a}{2\pi} t\cos^2 t + \frac{R^2a}{2\pi}\sin t\cos t\right]dt$$

$$= \frac{R^2a}{2\pi}\int_0^{2\pi}\left[-t(\cos^2 t - \sin^2 t) + \frac{1}{2}\sin 2t\right]dt$$

$$= \frac{R^2a}{2\pi}\int_0^{2\pi}\left[t\cos 2t + \frac{1}{2}\sin 2t\right]dt = \frac{R^2a}{2\pi}\left[\frac{t\sin 2t}{2}\Big|_0^{2\pi} - \frac{1}{2}\int_0^{2\pi}\sin 2t\,dt\right.$$

$t = u$ $\quad \cos 2t\,dt = dv$

$dt = du$ $\quad \frac{\sin 2t}{2} = v$

$$\left. + \frac{1}{2}\int_0^{2\pi}\sin 2t\,dt\right] = \boxed{0}$$