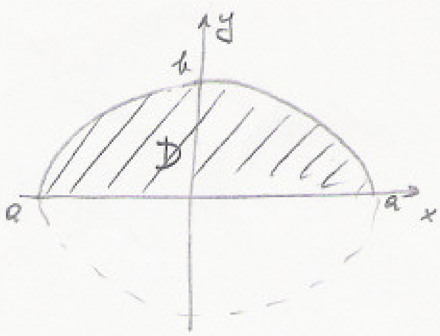
DOLOČI TEŽIŠČE LIKA, KI GA DOLOČATA NEENAČBI

DN 3

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1 \quad \text{IN} \quad y\ge 0$$



UVEDEMO POSPLOŠENE POLARNE KOORDINATE

$$x = a r\cos\varphi$$
$$y = b r\sin\varphi$$
$$J = abr$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1$$

$$\frac{\cancel{a^2} r^2\cos^2\varphi}{\cancel{a^2}}+\frac{\cancel{b^2} r^2\sin^2\varphi}{\cancel{b^2}}\le 1$$

$$r^2\le 1$$

$$m=\iint_D dx\,dy=\int_0^{\pi} d\varphi\int_0^1 \underbrace{abr}_{J}\,dr = ab\pi\left[\frac{r^2}{2}\right]_0^1=\boxed{\frac{ab\pi}{2}}$$

$$\iint_D x\,dx\,dy=\int_0^{\pi} d\varphi\int_0^1 ar\cos\varphi\;\underbrace{abr}_{J}\,dr = a^2b\left[\sin\varphi\right]_0^{\pi}\left[\frac{r^3}{3}\right]_0^1=\boxed{0}$$

$\Rightarrow$ $\boxed{x^*=0}$ (TO JE OČITNO TUDI BREZ RAČUNA, SAJ JE LIK HOMOGEN IN SIMETRIČEN GLEDE NA OS y)

$$\iint_D y\,dx\,dy=\int_0^{\pi} d\varphi\int_0^1 br\sin\varphi\;\underbrace{abr}_{J}\,dr = ab^2\left[-\cos\varphi\right]_0^{\pi}\left[\frac{r^3}{3}\right]_0^1=\boxed{\frac{2ab^2}{3}}$$

$$\Rightarrow\quad y^*=\frac{\iint_D y\,dx\,dy}{m}=\frac{\frac{2ab^2}{3}}{\frac{ab\pi}{2}}=\boxed{\frac{4b}{3\pi}}$$

TEŽIŠČE $T\left(0,\frac{4b}{3\pi}\right)$