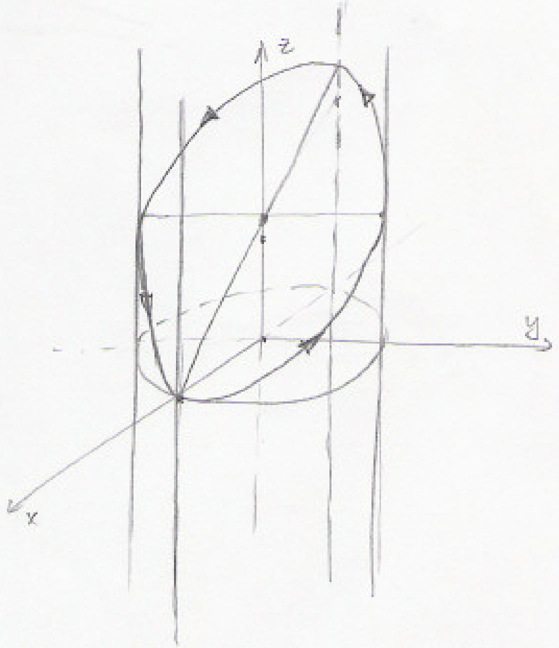
IZRAČUNAJ $\oint_\gamma (y-z)\,dx + (z-x)\,dy + (x-y)\,dz$, KJER

JE γ PRESEK VALJA $x^2+y^2=1$ IN RAVNINE $x+z=1$



ORIENTIRANO V SMERI NASPROTNI SMERI URINEGA KAZALCA, ČE GLEDAMO IZ $T(0,0,10)$

KER JE PROJEKCIJA KROG S POLMEROM 1

JE $x = 1\cdot\cos t$ $\quad t\in[0,2\pi]$

$y = 1\cdot\sin t$

IN

$z = 1 - x = 1-\cos t$

$$\vec{p}(t) = (\overbrace{\cos t}^{x}, \overbrace{\sin t}^{y}, \overbrace{1-\cos t}^{z})$$

$$\dot{\vec{p}}(t) = (\underbrace{-\sin t}_{\dot{x}}, \underbrace{\cos t}_{\dot{y}}, \underbrace{\sin t}_{\dot{z}})$$

$$\oint_\gamma (y-z)\,dx + (z-x)\,dy + (x-y)\,dz =$$

$$= \int_0^{2\pi} \Big[\underbrace{(\sin t - 1 + \cos t)}_{y-x}\underbrace{(-\sin t)\,dt}_{dx} + \underbrace{(1-\cos t - \cos t)}_{z-x}\underbrace{\cos t\,dt}_{dy} + \underbrace{(\cos t - \sin t)}_{x-y}\underbrace{\sin t\,dt}_{dz}$$

$$= \int_0^{2\pi}\left[-\sin^2 t + \sin t - \cancel{\cos t\sin t} + \cos t - 2\cos^2 t + \cancel{\cos t\sin t} - \sin^2 t\right] dt$$

$$= \int_0^{2\pi}\left[-2 + \sin t + \cos t\right] dt = -2\int_0^{2\pi} dt + \underbrace{\int_0^{2\pi}\sin t\,dt}_{0} + \underbrace{\int_0^{2\pi}\cos t\,dt}_{0} = \boxed{-4\pi}$$