IZRAČUNAJTE

$$\int_{K:A}^{B} 2x\cos z\, dx + z\, dy + (y - x^2 \sin z)\, dz$$

VZDOLŽ

$$\vec{K} : \vec{r}(t) = (t \sin t, t - \cos t, \cos^2 t) \quad , \quad t \in [0, \tfrac{\pi}{2}]$$

NAMIG: POGLEJTE, ČE JE POD INTEGRALOM POPOLNI DIFERENCIAL

$$A(0, -1, 1) \quad , B\left(\tfrac{\pi}{2}, \tfrac{\pi}{2}, 0\right)$$

ALI OBSTAJA TAK u, DA JE $\operatorname{grad} u = (2x\cos z, z, y - x^2 \sin z)$

$u_x \overset{*}{=} 2x\cos z$

$u_y \overset{**}{=} z$

$u_z \overset{***}{=} y - x^2 \sin z$

$u_x \overset{*}{=} 2x\cos z \implies u \overset{(1)}{=} x^2\cos z + \varphi(y, z)$

$\implies u_y = \varphi_y(y, z)$

$u_y \overset{**}{=} z$

$\implies$

$\implies \varphi(y, z) = yz + \psi(z)$

$\overset{(1)}{\implies} u \overset{(2)}{=} x^2\cos z + yz + \psi(z)$

$\implies u_z = -x^2 \sin z + y + \psi'(z)$

$u_z \overset{***}{=} y - x^2 \sin z$

$\implies \psi'(z) = 0 \implies \psi(z) = C$

$\overset{(2)}{\implies} u = x^2\cos z + yz + C$

Preizkus: $\operatorname{grad} u = (2x\cos z, z, -x^2\sin z + y)$

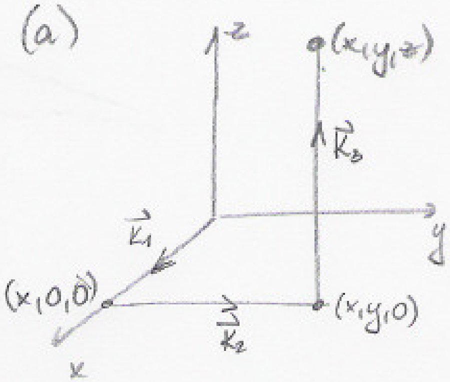
$$\int_{K:A}^{B} 2x\cos z\, dx + z\, dy + (y - x^2\sin z)\, dz = \int_{K:A}^{B} du = u(B) - u(A)$$

$$= u\left(\tfrac{\pi}{2}, \tfrac{\pi}{2}, 0\right) - u(0, -1, 1) = \left(\tfrac{\pi}{2}\right)^2 \cdot 1 + \tfrac{\pi}{2} \cdot 0 + C - 0^2\cos 1 - (-1)\cdot 1 - ($$

$= \pi^2 \ldots 1$

POSKUSIMO POTENCIAL DOBITI ŠE S KRIVULJNIM INTEGRALOM

(a)



$$\vec{U} = (2x\cos z,\ z,\ y - x^2 \sin z)$$

$$\int_{\vec{K}} \vec{U}\, d\vec{r} = \int_{\vec{K}_1+\vec{K}_2+\vec{K}_3} \vec{U}\, d\vec{r} = x^2 + 0 + \underbrace{yz + x^2\cos z - x^2}$$

$$= \boxed{yz + x^2 \cos z}$$

$\vec{K}_1:\quad \vec{r}(t) = (tx, 0, 0) \qquad t \in [0,1]$

$\quad\ \ \dot{\vec{r}}(t) = (x, 0, 0)$

$$\int_{\vec{K}_1} \vec{U}\, d\vec{r} = \int_0^1 (2tx\cos 0,\ 0,\ 0 - x^2t^2 \sin 0)(x,0,0)\, dt$$

$$= 2x^2 \int_0^1 t\, dt = \frac{\cancel{2}x^2t^2}{\cancel{2}}\Big|_0^1 = x^2 \qquad (*)$$

$\vec{K}_2:\quad \vec{r}(t) = (x, ty, 0) \qquad t \in [0,1]$

$\quad\ \ \dot{\vec{r}}(t) = (0, y, 0)$

$$\int_{K_2} \vec{U}\, d\vec{r} = \int_0^1 (2x\cos 0,\ 0,\ ty - x^2 \sin 0)(0,y,0)\, dt = 0 \qquad (**)$$

$\vec{K}_3:\quad \vec{r}(t) = (x, y, tz) \qquad t \in [0,1]$

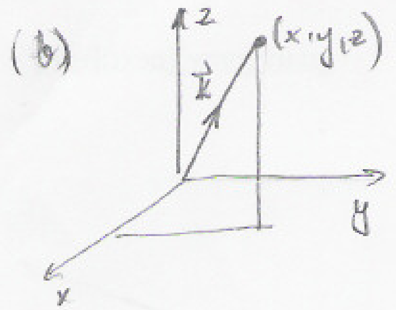
$\quad\ \ \dot{\vec{r}}(t) = (0, 0, z)$

$$\int_{\vec{K}_3} \vec{U}\, d\vec{r} = \int_0^1 (2x\cos zt,\ zt,\ y - x^2 \sin zt)(0,0,z)\, dt$$

$$= \int_0^1 (yz - x^2 z \sin zt)\, dt = yzt\Big|_0^1 - x^2\cancel{z}\left(\frac{-\cos zt}{\cancel{z}}\right)\Big|_0^1$$

$$= yz + x^2\cos z - x^2 \qquad (***)$$

(b)



$$\vec{r}(t) = (xt, yt, zt) \qquad t \in [0,1]$$
$$\dot{\vec{r}}(t) = (x, y, z)$$

$$\int_{\vec{K}} \vec{v}\, d\vec{r} = \int_0^1 (2xt\cos zt,\ zt,\ yt - x^2t^2\sin zt)(x,y,z)\,dt$$

$$= 2x^2\int_0^1 t\cos zt\,dt + yz\int_0^1 t\,dt + yz\int_0^1 t\,dt - x^2z\int_0^1 t^2\sin zt\,dt$$

$t = u$ * $\quad \cos zt\,dt = dv \qquad\qquad t^2 = u \quad \sin zt\,dt = dv$

$dt = du \quad \frac{\sin zt}{z} = v \qquad\qquad 2t\,dt = du \quad -\frac{\cos zt}{z} = v$

$$= 2x^2\left(\frac{t\sin zt}{z}\Big|_0^1 - \frac{1}{z}\int_0^1 \sin zt\,dt\right) + 2yz\int_0^1 t\,dt - x^2z\left(-\frac{t^2\cos zt}{z}\Big|_0^1 + \frac{2}{z}\int_0^1 t\cos zt\,dt\right)$$ *

$$= 2x^2\underbrace{\left(\frac{\sin z}{z} + \frac{1}{z}\,\frac{\cos zt}{z}\Big|_0^1\right)}_{*} + \cancel{2}yz\frac{t^2}{\cancel{2}}\Big|_0^1 - x^2z\left(-\frac{\cos z}{z} + \frac{2}{z}\left(\frac{\sin z}{z} + \frac{1}{z}\,\frac{\cos zt}{z}\Big|_0^1\right)\right)$$

$$= \cancel{\frac{2x^2}{z}\sin z} + \cancel{\frac{2x^2}{z^2}\cos z} - \cancel{\frac{2x^2}{z^2}} + yz + x^2\cos z - \cancel{\frac{2x^2}{z}\sin z} - \cancel{\frac{2x^2}{z^2}\cos z} + \cancel{\frac{2x^2}{z^2}}$$

$$= \underline{yz + x^2\cos z}$$

* ... ISTI INTEGRAL SE JE POJAVIL ŽE PREJ, ZATO GA NAM NI TREBA ŠE ENKRAT IZRAČUNAT