D.N.

DN 34

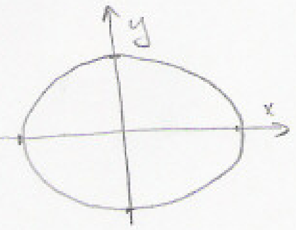
IZRAČUNAJ $\oint_{\vec{\gamma}} (xy+x+y)\,dx + (xy+x-y)\,dy$, KJER JE

$\vec{\gamma}$ POZITIVNO ORIENTIRANA ELIPSA $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(a) NEPOSREDNO

(b) Z UPORABO GREENOVE FORMULE $\boxed{\oint_{\partial\Omega} P\,dx + Q\,dy = \iint_{\Omega}\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy}$

(a)



$x = a\cos\varphi \Rightarrow dx = -a\sin\varphi\,d\varphi \qquad \varphi\in[0,2\pi]$

$y = a\sin\varphi \Rightarrow dy = b\cos\varphi\,d\varphi$

$\frac{x^2}{a^2}+\frac{y^2}{b^2} = \frac{\cancel{a^2}\cos^2\varphi}{\cancel{a^2}} + \frac{\cancel{b^2}\sin^2\varphi}{\cancel{b^2}} = \cos^2\varphi+\sin^2\varphi = 1$

$$\oint_{\vec{\gamma}} (xy+x+y)\,dx + (xy+x-y)\,dy =$$

$$= \int_0^{2\pi}\left[(ab\cos\varphi\sin\varphi + a\cos\varphi + b\sin\varphi)(-a\sin\varphi) + (ab\cos\varphi\sin\varphi + a\cos\varphi - b\sin\varphi)\,b\cos\varphi\right]d\varphi$$

$$= \int_0^{2\pi}\left[-a^2b\cos\varphi\sin^2\varphi - a^2\cos\varphi\sin\varphi - ab\sin^2\varphi + ab^2\cos^2\varphi\sin\varphi + ab\cos^2\varphi - b^2\cos\varphi\sin\varphi\right]d\varphi$$

$$= \underbrace{-a^2b\int_0^{2\pi}\cos\varphi\sin^2\varphi\,d\varphi}_{0,\ \text{KER JE EKSP. PRI COS LIH}} - \underbrace{(a^2+b^2)\int_0^{2\pi}\cos\varphi\sin\varphi}_{0,\ \text{KER STA OBA EKSP. LIHA}} - ab\int_0^{2\pi}\sin^2\varphi\,d\varphi + ab\int_0^{2\pi}\cos^2\varphi\,d\varphi + \underbrace{ab^2\int_0^{2\pi}\cos^2\varphi\sin\varphi\,d\varphi}_{0,\ \text{KER JE EKS. PRI SIN LIH}}$$

$$= -4ab\int_0^{\frac{\pi}{2}}\sin^2\varphi\,d\varphi + 4ab\int_0^{\frac{\pi}{2}}\cos^2\varphi\,d\varphi = -2ab\,B\left(\tfrac{3}{2},\tfrac{1}{2}\right) + 2ab\,B\left(\tfrac{1}{2},\tfrac{3}{2}\right) = \boxed{0}$$

(b) $\oint_{\vec{\gamma}} \underbrace{(xy+x+y)}_{P}dx + \underbrace{(xy+x-y)}_{Q}dy = \iint_{\Omega}\left[(y+1)-(x+1)\right]dx\,dy = \iint_{\Omega}[y-x]\,dx\,dy$

$P = xy+x+y \Rightarrow P_y = x+1$

$Q = xy+x-y \Rightarrow Q_x = y+1$

$x = ar\cos\varphi$

$y = br\sin\varphi$

$J = \begin{vmatrix} a\cos\varphi & -ar\sin\varphi \\ b\sin\varphi & br\cos\varphi \end{vmatrix} = abr$

$\frac{x^2}{a^2}+\frac{y^2}{b^2} = r^2 = 1$

$$= \int_0^{2\pi}d\varphi\int_0^1 \underbrace{[br\sin\varphi - ar\cos\varphi]}_{y-x}\ \underbrace{abr}_{J} = ab^2\underbrace{\int_0^{2\pi}\sin\varphi\,d\varphi}_{0}\int_0^1 r^2\,dr - a^2b\underbrace{\int_0^{2\pi}\cos\varphi\,d\varphi}_{0}\int_0^1 r^2\,dr$$

(LIH EKS. PRI sin) (LIH EKS. PRI COS)

$= \boxed{0}$