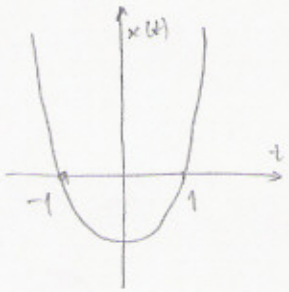
NARIŠI KRIVULJO IN IZRAČUNAJ PLOŠČINO ZANKE

$$\begin{cases} x(t) = t^2 - 1 \\ y(t) = t^3 - t \end{cases}$$



$x(t)$: $D_{x(t)} = \mathbb{R}$

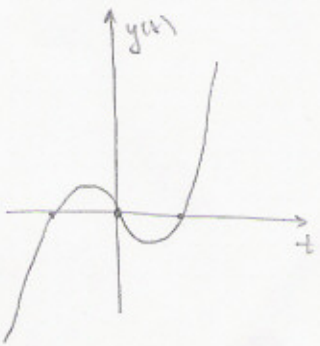
N : $t_1 = 1$

$t_2 = -1$

$\dot{x} = 2t \Rightarrow$ EKSTREM PRI $t=0$, $x(0) = -1$

$y(t)$: $D_{y(t)} = \mathbb{R}$

N : $t_1 = 0$ $|t_2 = 1,\ t_3 = -1$



$\dot{y} = 3t^2 - 1 = 0 \Rightarrow t_{1/2} = \pm\frac{\sqrt{3}}{3}$

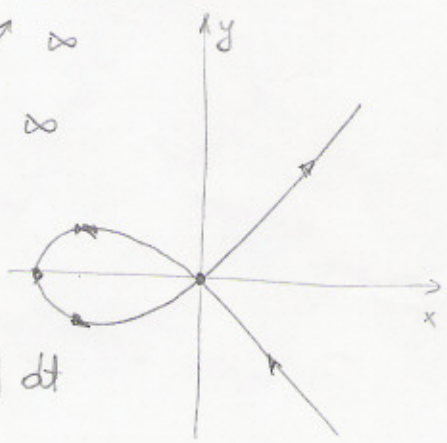
$y\left(\frac{\sqrt{3}}{3}\right) = \pm\frac{\sqrt{3}}{3}\left(\frac{1}{3} - 1\right) = \mp\frac{2\sqrt{3}}{9}$

$$y'(x) = \frac{\dot{y}}{\dot{x}} = \frac{3t^2-1}{2t} = \varphi \Rightarrow \dot{\varphi} = \frac{6t(2t) - (3t^2-1)\cdot 2}{4t^2} = \frac{12t^2 - 6t^2 + 2}{4t^2} = \frac{6t^2+2}{4t^2}$$

$$= \frac{3t^2+1}{2t^2}$$

$$y''(x) = \frac{\dot{\varphi}}{\dot{x}} = \frac{3t^2+1}{2t^2 \cdot 2t} = \frac{3t^2+1}{4t^3} > 0 \quad \text{za } t > 0$$

t	$-\infty$		-1		$-\frac{\sqrt{3}}{3}$		0		$\frac{\sqrt{3}}{3}$		1		∞
x	∞	↘	0	↘	$-\frac{2}{3}$	↘	-1	↗	$\frac{2}{3}$	↗	0	↗	∞
y	$-\infty$	↗	0	↗	$\frac{2\sqrt{3}}{9}$	↘	0	↘	$-\frac{2\sqrt{3}}{9}$	↗	0	↗	∞



$$pl = \frac{1}{2}\int_{-1}^{1}[x\dot{y} - y\dot{x}]\,dt = \frac{1}{2}\int_{-1}^{1}[(t^2-1)(3t^2-1) - (t^3-t)2t]\,dt$$

$$= \frac{1}{2}\int_{-1}^{1}[3t^4 - 3t^2 - t^2 + 1 - 2t^4 + 2t^2]\,dt = \frac{1}{2}\int_{-1}^{1}[t^4 - 2t^2 + 1]\,dt = \left[\frac{t^5}{5} - \frac{2t^3}{3} + 1\right]_0^1$$

SODA F.

$$= \frac{1}{5} - \frac{2}{3} + 1 = \frac{3 - 10 + 15}{15} = \boxed{\frac{8}{15}}$$