POIŠČI ENAČBE TANGENTNIH RAVNIN IN NORMAL NA DANE PLOSKVE V DANI TOČKI

(a) $z(x,y) = x^2 - xy - y^2$ v točki $T(1,1,-1)$

$p = z_x = 2x - y$ $\quad p(1,1) = 2-1 = 1$

$q = z_y = -x - 2y$ $\quad q(1,1) = -1-2 = -3$

$\Rightarrow \vec{n} = (p,q,-1) = (1,-3,-1)$

~~$\vec{n}(1,1) = (1,-3,-1)$~~

$\vec{n} = (p,q,-1)$

$= (1,-3,-1)$

TANGENTNA RAVNINA:

$$\vec{r}\,\vec{n} = \vec{r}_T\,\vec{n}$$

$$(x,y,z)(1,-3,-1) = (1,1,-1)(1,-3,-1)$$

$$x - 3y - z = 1 - 3 + 1 = -1$$

$$\underline{x - 3y - z = -1}$$

NORMALA $\vec{s} = \vec{n} = (1,-3,-1)$

$$\frac{x-1}{1} = \frac{y-1}{-3} = \frac{z+1}{-1}$$

$$\underline{x - 1 = \frac{1-y}{3} = -z - 1}$$

ALI

$$\vec{r} = \vec{r}_T + \lambda\vec{s} \qquad \lambda \in \mathbb{R}$$

$$\underline{\vec{r} = (1,1,-1) + \lambda(1,-3,-1), \ \lambda \in \mathbb{R}}$$

ALI

$$\begin{aligned} x &= 1 + \lambda \\ y &= 1 - 3\lambda \\ z &= -1 - \lambda \end{aligned} \quad , \lambda \in \mathbb{R}$$

(6) $(z^2-x^2)xyz - y^5 = 5$ V TOČKI $T(1,1,2)$

$$F(x,y,z) = (z^2-x^2)xyz - y^5 - 5$$
$$= xyz^3 - x^3yz - y^5 - 5 \equiv 0$$

$F_x = yz^3 - 3x^2yz$ $\quad F_x(1,1,2) = 1\cdot 8 - 3\cdot 1\cdot 1\,2 = 2$

$F_y = xz^3 - x^3z - 5y^4$ $\quad F_y(1,1,2) = 1\cdot 8 - 1\cdot 2 - 5\cdot 1 = 1$

$F_z = 3xyz^2 - x^3y$ $\quad F_z(1,1,2) = 3\cdot 1\cdot 1\cdot 4 - 1\cdot 1 = 11$

$\Rightarrow$

$\Rightarrow (\text{grad}F)(1,1,2) = (2,1,11) = \vec{n}$

TANGENTNA RAVNINA

$$\vec{r}\,\vec{n} = \vec{r}_T\,\vec{n}$$
$$2x + y + 11z = (1,1,2)(2,1,11) = 2+1+22 = 25$$

$$\underline{2x + y + 11z = 25}$$

NORMALA

$$\vec{s} = \vec{n} = (2,1,11)$$

$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{11}$$

$$\underline{\frac{x-1}{2} = y-1 = \frac{z-2}{11}} \quad \text{ALI} \quad \underline{\vec{r} = (1,1,2) + \lambda(2,1,11),\ \lambda\in\mathbb{R}}$$

ALI $\quad x = 1+2\lambda$
$y = 1+\lambda$, $\lambda \in \mathbb{R}$
$z = 2+11\lambda$

(c) $z = y + \ln\frac{x}{z}$ V TOČKI $T(1,1,1)$

$$F(x,y,z) = y + \ln\frac{x}{z} - z = 0$$

$$F_x = \frac{z}{x \cdot z} = \frac{1}{x} \qquad F_x(1,1,1) = 1$$

$$F_y = 1 \qquad F_y(1,1,1) = 1$$

$$F_z = \frac{z(-1)x}{x \cdot z^2} - 1 = -\frac{1}{z} - 1 \qquad F_z(1,1,1) = -2$$

$$\Big\} \vec{n} = (1,1,-2)$$

TANGENTNA RAVNINA

$$\vec{r}\,\vec{n} = \vec{r}_T\vec{n}$$

$$(x,y,z)(1,1,-2) = (1,1,1)(1,1,-2)$$

$$x + y - 2z = 1 + 1 - 2 = 0$$

$$\underline{\underline{x + y - 2z = 0}}$$

NORMALA:

$$\vec{s} = \vec{n} = (1,1,-2)$$

$$\frac{x-1}{1} = \frac{y-1}{1} = \frac{z+2}{-2}$$

$$\underline{\underline{x-1 = y-1 = \frac{z+2}{-2}}}$$

ALI

$$\vec{r} = \vec{r}_T + \lambda\vec{s} \quad , \lambda \in \mathbb{R}$$

$$\underline{\underline{\vec{r} = (1,1,1) + \lambda(1,1,-2) \, , \lambda \in \mathbb{R}}}$$

(c) $z = \sqrt{x^2+y^2} - xy$ v TOČKI $T(3,4,-7)$

$$p = z_x = \frac{2x}{2\sqrt{x^2+y^2}} - y = \frac{x}{\sqrt{x^2+y^2}} - y \Rightarrow p(3,4) = \frac{3}{5} - 4 = \frac{-17}{5}$$

$$q = z_y = \frac{y}{\sqrt{x^2+y^2}} - x \rightarrow q(3,4) = \frac{4}{5} - 3 = \frac{-11}{5}$$

$$\Rightarrow \vec{n} \parallel (p, q, -1) = \left(\frac{-17}{5}, \frac{-11}{5}, -1\right) = \frac{-1}{5}(17, 11, 5)$$

ZA NORMALO LAHKO VZAMEMO $\vec{n} = (17, 11, 5)$

TANGENTNA RAVNINA

$$\vec{r}\,\vec{n} = \vec{r}_T\,\vec{n}$$

$$17x + 11y - 5z = (3,4,-7)(17,11,5)$$

$$= 51 + 44 - 35 = 60$$

$$\underline{17x + 11y + 5z = 60}$$

NORMALA:

$$\vec{s} = \vec{n} = (17, 11, 5)$$

$$\underline{\frac{x-3}{17} = \frac{y-4}{11} = \frac{z+7}{5}}$$

ALI

$$\boxed{\begin{aligned} x &= 3 + 17\lambda \\ y &= 4 + 11\lambda \\ z &= -7 + 5\lambda \end{aligned} \quad , \lambda \in \mathbb{R}}$$

(d) $\vec{f}(r,\varphi) = (r\cos\varphi, r\sin\varphi, r) \quad , r \in [0,2], \varphi \in [0, 2\pi]$

V TOČKI $T(\sqrt{2}, \sqrt{2}, 2)$

$(r\cos\varphi, r\sin\varphi, r) = (\sqrt{2}, \sqrt{2}, 2) \Rightarrow \underline{r = 2}$

$\left.\begin{aligned} \cos\varphi &= \frac{\sqrt{2}}{2} \\ \sin\varphi &= \frac{\sqrt{2}}{2} \end{aligned}\right\} \underline{\varphi = \frac{\pi}{4}}$

$\left.\begin{aligned} \vec{f}_r &= (\cos\varphi, \sin\varphi, 1) \\ \vec{f}_\varphi &= (-r\sin\varphi, r\cos\varphi, 0) \end{aligned}\right\} \Rightarrow$

$$\vec{f}_r \times \vec{f}_\varphi = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\varphi & \sin\varphi & 1 \\ -r\sin\varphi & r\cos\varphi & 0 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ \cos\varphi & \sin\varphi \\ -r\sin\varphi & r\cos\varphi \end{matrix}$$

$$= (-r\cos\varphi, -r\sin\varphi, r)$$

$$(\vec{f}_r \times \vec{f}_\varphi)\left(2, \frac{\pi}{4}\right) = \left(-2\cdot\frac{\sqrt{2}}{2}, -2\frac{\sqrt{2}}{2}, 2\right)$$

$$= (-\sqrt{2}, -\sqrt{2}, 2)$$

$$= -\sqrt{2}(1, 1, -\sqrt{2}) \Rightarrow \vec{n} = (1, 1, -\sqrt{2})$$

TANGENTNA RAVNINA:

$$x + y - \sqrt{2}z = (1, 1, -\sqrt{2})(\sqrt{2}, \sqrt{2}, 2)$$

$$= \sqrt{2} + \sqrt{2} - 2\sqrt{2} = 0$$

$$\underline{x + y - \sqrt{2}z = 0}$$

NORMALA

$$\left.\begin{aligned} x &= \sqrt{2} + \lambda \\ y &= \sqrt{2} + \lambda \\ z &= \sqrt{2} - \sqrt{2}\lambda \end{aligned}\right| \quad , \lambda \in \mathbb{R}$$