POIŠČI TISTE TANGENTNE RAVNINE PLOSKVE

$$4x^2+6y^2+4z^2+4xz-8y-4z+3=0,$$

KI SO VZPOREDNE RAVNINI $\Pi: x+2y+7=0$

$F_x = 8x+4z$

$F_y = 12y-8$

$F_z = 8z+4x-4$

$$(\text{grad}F)(a,b,c) = (8a+4c, 12b-8, 8c+4a-4)$$
$$= 4(2a+c, 3b-2, 2c+a-1)$$

ENAČBA TANG. RAVNINE Σ V $T(a,b,c)$

$$(2a+c)x + (3b-2)y + (2c+a-1)z = (2a+c, 3b-2, 2c+a-1)(a,b,c)$$
$$= 2a^2+ac+3b^2-2b+2c^2+ac-1$$
$$= 2a^2+3b^2+2c^2+2ac-2b-1$$

$\vec{n}_\Sigma (a,b,c) = (2a+c, 3b-2, 2c+a-1)$

$\vec{n}_\Pi = (1,2,0)$

$\Rightarrow$ KER $\vec{n}_\Sigma \parallel \vec{n}_\Pi$, JE $\vec{n}_\Sigma \times \vec{n}_\Pi = \vec{0}$

$$\vec{n}_\Sigma \times \vec{n}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2a+c & 3b-2 & 2c+a-1 \\ 1 & 2 & 0 \end{vmatrix} = (-4c-2a+2, 2c+a-1, 4a+2c-3b+2)$$

$$\Rightarrow \quad -4c-2a+2=0 \qquad a+2c=1$$
$$2c+a-1=0 \qquad a+2c=1$$
$$4a+2c-3b+2=0 \qquad 4a-3b+2c=-2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 4 & -3 & 2 & -2 \end{array}\right] \cdot(-4) \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & -3 & -6 & -6 \end{array}\right] \cdot\left(-\tfrac{1}{3}\right) \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 2 \end{array}\right] \Rightarrow a = 1-2c$$
$$\Rightarrow b = 2-2c$$

KDE $(a,b,c) \in P$, VELJÁ

$$4(1-2c)^2 + 6(2-2c)^2 + 4c^2 + 4(1-2c)\cdot c - 8(2-2c) - 4c + 3 = 0$$

$$4(1-4c+4c^2) + 6(4-8c+4c^2) + 4c^2 + 4c - 8c^2 - 16 + 16c - 4c + 3 = 0$$

$$4 - 16c + 16c^2 + 24 - 48c + 24c^2 + 4c^2 + 4c - 8c^2 - 16 + 16c - 4c + 3 = 0$$

$$36c^2 - 48c + 15 = 0$$

$$12c^2 - 16c + 5 = 0 \Rightarrow c_{1,2} = \frac{16 \pm \sqrt{16^2 - 4\cdot 12\cdot 5}}{24} = \frac{16 \pm 4\sqrt{16-15}}{24}$$

$$= \frac{4 \pm 1}{6} \Rightarrow c_1 = \frac{1}{2}$$

$$c_2 = \frac{5}{6}$$

(a) $c_1 = \frac{1}{2} \Rightarrow a_1 = 1 - 2c_1 = 0$

$b_1 = 2 - 2c_1 = 2 - 1 = 1$

$\Big\}$ $T_1(0,1,\frac{1}{2})$

$\vec{n_2} \parallel (1,2,0)$

$1\cdot x + 2y + 0\cdot z = (1,2,0)(0,1,\frac{1}{2})$

$$x + 2y = 2$$

(b) $c_2 = \frac{5}{6} \Rightarrow a_1 = 1 - 2\cdot\frac{5}{6} = 1 - \frac{5}{3} = -\frac{2}{3}$

$b_1 = 2 - 2\cdot\frac{5}{6} = \frac{1}{3}$

$\Big\}$ $T_2(-\frac{2}{3}, \frac{1}{3}, \frac{5}{6})$

$1\cdot x + 2\cdot y + 0\cdot z = (1,2,0)(-\frac{2}{3}, \frac{1}{3}, \frac{5}{6})$

$$x + 2y = 0$$