5N.

POKAŽI, DA TANGENTNA RAVNINA NA PLOSKEV $xyz = a^3$ V POLJUBNI TOČKI TVORI S KOORDINATNIMI RAVNINAMI TETRAEDER KONSTANTNEGA VOLUMNA.

$$F(x,y,z) = xyz - a^3 = 0$$

$$F_x = yz$$
$$F_y = xz$$
$$F_z = xy$$

$$(\text{grad}\, F)(x_0,y_0,z_0) = (y_0z_0, x_0z_0, x_0y_0)$$

ENAČBA TANGENTNE RAVNINE V TOČKI (x_0,y_0,z_0)

$$y_0z_0x + x_0z_0y + x_0y_0z = (y_0z_0, x_0z_0, x_0y_0)(x_0,y_0,z_0)$$
$$y_0z_0x + x_0z_0y + x_0y_0z = 3x_0y_0z_0$$

ODSEKI NA KOORDINATNIH OSEH:

OS x: $y=0, z=0 \Rightarrow y_0z_0x + x_0z_0\cdot 0 + x_0y_0\cdot 0 = 3x_0y_0z_0$

$$x = 3x_0$$

OS y: $x=0, z=0 \Rightarrow y = 3y_0$

OS z: $x=0, y=0 \Rightarrow z = 3z_0$

$$V_\Delta = \frac{1}{6}(3x_0)\cdot(3y_0)(3z_0) = \frac{9}{2}x_0y_0z_0 = \frac{9}{2}a^3 \Rightarrow$$

a^3 (KER JE $T(x_0,y_0,z_0)$ NA PLOSKVI $xyz = a^3$)

$\Rightarrow V_\Delta$ JE KONSTANTEN V VSAKI TOČKE PLOSKVE