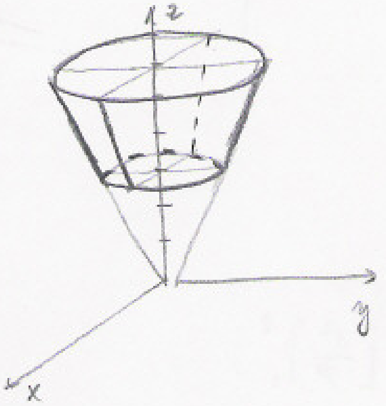
IZRAČUNAJTE POVRŠINO PLAŠČA STOŽCA

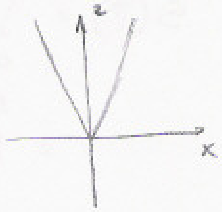
$$P = \{ (x,y,z) \in \mathbb{R}^3 ; \quad z = 3\sqrt{x^2+y^2} \ , 3 \le z \le 6 \}$$



$z=3: \ \sqrt{x^2+y^2} = 1 \Rightarrow x^2+y^2 = 1$

$z=6: \ \sqrt{x^2+y^2} = 2 \Rightarrow x^2+y^2 = 4$

$y=0: \quad z = 3|x|$



1\. pot: $\quad z = 3\sqrt{x^2+y^2}$

$$\left.\begin{aligned} p &= z_x = \frac{3 \cdot 2x}{2\sqrt{x^2+y^2}} = \frac{3x}{\sqrt{x^2+y^2}} \\ q &= z_y = \frac{3y}{\sqrt{x^2+y^2}} \end{aligned}\right\} \quad 1 + p^2 + q^2 = 1 + \frac{9x^2}{x^2+y^2} + \frac{9y^2}{x^2+y^2} = \frac{10(x^2+y^2)}{x^2+y^2} = 10$$

$$S(P) = \iint_P \sqrt{1+p^2+q^2}\, dx\, dy = \int_0^{2\pi} d\varphi \int_1^2 \sqrt{10} \cdot r = \cancel{2}\pi\sqrt{10}\left[\frac{r^2}{\cancel{2}}\right]_1^2$$

$x = r\cos\varphi \qquad r \in [1,2], \ \varphi \in [0, 2\pi]$
$y = r\sin\varphi$
$J = r$

$$= \sqrt{10}\pi\,(4-1) = \underline{\underline{3\sqrt{10}\,\pi}}$$

2\. pot

$\vec{f}(r,\varphi) = (r\cos\varphi, r\sin\varphi, 3r) \qquad r \in [1,2], \ \varphi \in [0,2\pi]$

$\vec{f}_r = (\cos\varphi, \sin\varphi, 3)$

$\vec{f}_\varphi = (-r\sin\varphi, r\cos\varphi, 0)$

$$\left.\begin{aligned} E &= \vec{f}_r \cdot \vec{f}_r = \cos^2\varphi + \sin^2\varphi + 9 = 10 \\ F &= \vec{f}_r \cdot \vec{f}_\varphi = -r\cos\varphi\sin\varphi + r\sin\varphi\cos\varphi + 0 = 0 \\ G &= \vec{f}_\varphi \cdot \vec{f}_\varphi = r^2\sin^2\varphi + r^2\cos^2\varphi = r^2 \end{aligned}\right\} \sqrt{EG-F^2} = \sqrt{10}\, r$$

$$S(P) = \int_0^{2\pi} d\varphi \int_1^2 \sqrt{EG-F^2}\, r\, dr$$

$$= \int_0^{2\pi} d\varphi \int_1^2 \sqrt{10}\, r\, dr = \cancel{2}\pi\sqrt{10}\left[\frac{r^2}{\cancel{2}}\right]_1^2$$

$$= \boxed{3\sqrt{10}\,\pi}$$