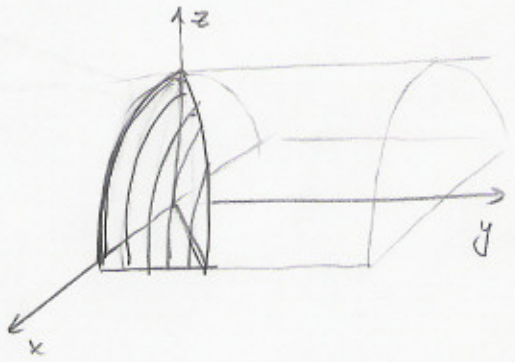
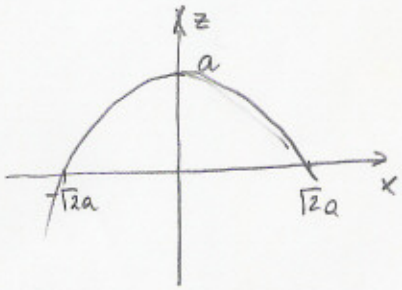
DOLOČITE POVRŠINO TISPEGA DELA PLOSKVE $x^2 = 2a(a-z)$, $a>0$, KI LEŽI MED RAVNINAMI $y=x$, $y=0$, $z=0$.

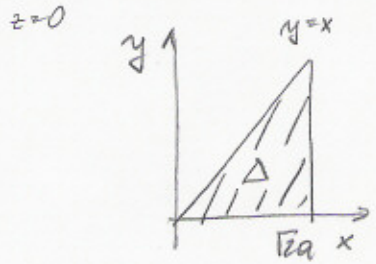


$$x^2 = 2a^2 - 2az$$

$$2az = 2a^2 - x^2$$

$$z = a - \frac{x^2}{2a}$$





$$S(\mathcal{P}) = \iint_{\mathcal{P}} dS = \iint_{\Delta} \sqrt{1+p^2+q^2}\, dx\,dy =$$

$$p = z_x = -\frac{2x}{2a} = -\frac{x}{a}$$
$$q = z_y = 0$$
$$\Big\} \quad 1+p^2+q^2 = 1+\frac{x^2}{a^2} = \frac{a^2+x^2}{a^2}$$

$$= \iint_{\Delta} \sqrt{\frac{a^2+x^2}{a^2}}\, dx\,dy = \frac{1}{a}\int_0^{\sqrt{2}a} dx \int_0^{x} \sqrt{a^2+x^2}\, dy =$$

$$= \frac{1}{a}\int_0^{\sqrt{2}a} x\sqrt{a^2+x^2}\, dx = \frac{1}{a}\int_a^{a\sqrt{3}} |t|\, t\, dt = \frac{1}{a}\left[\frac{t^3}{3}\right]_a^{a\sqrt{3}}$$

$a^2+x^2 = t^2$
$2x\,dx = 2t\,dt$

$x=0 \to t=a$
$x=\sqrt{2}a \to t^2 = a^2+2a^2 = 3a^2 \to t = a\sqrt{3}$

$$= \frac{1}{3a}\left[a^3 3\sqrt{3} - a^3\right] = \frac{a^2(3\sqrt{3}-1)}{3} = \boxed{\frac{a^2(3\sqrt{3}-1)}{3}}$$