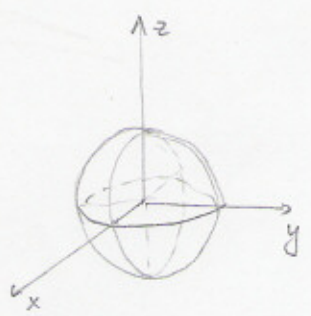


NAJ BO P SFERA  $x^2 + y^2 + z^2 = a^2$  IN GOSTOTA V VSAKI TOČKI SFERE NAJ BO BORAŽM ODDALJENOSTI TE TOČKE OD OSI Z. KOLIKSNA JE MASA SFERE? (REŠI PARAMETRIČNO)



$$\rho(x,y,z) = k\sqrt{x^2+y^2}$$

$$\vec{r}(\varphi, \psi) = (a\cos\varphi\cos\psi, a\sin\varphi\cos\psi, a\sin\psi) \quad \varphi \in [0, 2\pi], \psi \in [\frac{\pi}{2}, \frac{3\pi}{2}]$$

$$\vec{r}_\varphi = (-a\sin\varphi\cos\psi, a\cos\varphi\cos\psi, 0)$$

$$\vec{r}_\psi = (-a\cos\varphi\sin\psi, -a\sin\varphi\sin\psi, a\cos\psi)$$

$$E = \vec{r}_\varphi \cdot \vec{r}_\varphi = a^2\sin^2\varphi\cos^2\psi + a^2\cos^2\varphi\cos^2\psi = a^2\cos^2\psi(\sin^2\varphi + \cos^2\varphi) = a^2\cos^2\psi$$

$$F = \vec{r}_\varphi \cdot \vec{r}_\psi = a^2\sin\varphi\cos\varphi\sin\psi\cos\psi - a^2\sin\varphi\cos\varphi\sin\psi\cos\psi = 0$$

$$G = \vec{r}_\psi \cdot \vec{r}_\psi = a^2\cos^2\varphi\sin^2\psi + a^2\sin^2\varphi\sin^2\psi + a^2\cos^2\psi = a^2\sin^2\psi(\cos^2\varphi + \sin^2\varphi) + a^2\cos^2\psi = a^2(\sin^2\psi + \cos^2\psi) = a^2$$

$$\Rightarrow \sqrt{EG - F^2} = \sqrt{a^2\cos^2\psi a^2} = a^2|\cos\psi| = a^2\cos\psi$$

$$\rho(a\cos\varphi\cos\psi, a\sin\varphi\cos\psi, a\sin\psi) = \sqrt{a^2\cos^2\varphi\cos^2\psi + a^2\sin^2\varphi\cos^2\psi} = a|\cos\psi|\sqrt{\cos^2\varphi + \sin^2\varphi} = a\cos\psi$$

$$m = \iint_P \rho \, ds = \int_0^{2\pi} d\varphi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \underbrace{ka\cos\psi}_{\rho} \underbrace{a^2\cos\psi}_{\sqrt{EG-F^2}} d\psi = ka^3 \int_0^{2\pi} d\varphi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2\psi d\psi$$

$$= k a^3 2\pi \cdot 2 \cdot \frac{1}{2} B\left(\frac{1}{2}, \frac{3}{2}\right) = k a^3 \pi \frac{\Gamma(\frac{1}{2}) \Gamma(\frac{3}{2})}{1} = \underline{\underline{k a^3 \pi^2}}$$