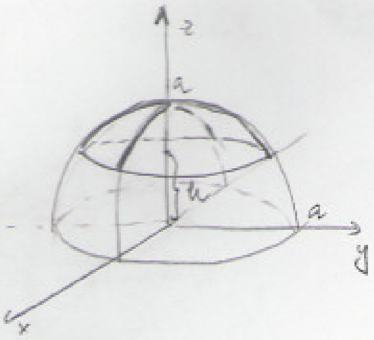
D.N.

ČI TEŽIŠČE PLOSKVE $x^2+y^2+z^2=a^2$ ZA $0<h<z<a$



ZARADI SIMETRIČNOSTI PLOSKVE JE $x_T=y_T=0$

PARAMETRIZACIJA PLOSKVE

$$\vec{f}(\varphi,\psi)=(a\cos\varphi\cos\psi,\ a\sin\varphi\cos\psi,\ a\sin\psi)$$

$z=a\sin\psi\leq h$

$\sin\psi\leq\frac{h}{a}$

$\psi\leq\arcsin\frac{h}{a}\Rightarrow$

$$\Rightarrow\ \varphi\in[0,2\pi),\ \psi\in\left[\arcsin\frac{h}{a},\frac{\pi}{2}\right]$$

$$\vec{f}_\varphi=(-a\sin\varphi\cos\psi,\ a\cos\varphi\cos\psi,\ 0)=a\cos\psi(-\sin\varphi,\cos\varphi,0)$$

$$\vec{f}_\psi=(-a\cos\varphi\sin\psi,\ -a\sin\varphi\sin\psi,\ a\cos\psi)=a(-\cos\varphi\sin\psi,-\sin\varphi\sin\psi,\cos\psi)$$

$$E=\vec{f}_\varphi\cdot\vec{f}_\varphi=a^2\cos^2\psi(\sin^2\varphi+\cos^2\varphi+0)=a^2\cos^2\psi$$

$$F=\vec{f}_\varphi\cdot\vec{f}_\psi=a^2\cos\psi(\sin\varphi\cos\varphi\sin\psi-\cos\varphi\sin\varphi\sin\psi)=0$$

$$G=\vec{f}_\psi\vec{f}_\psi=a^2(\cos^2\varphi\sin^2\psi+\sin^2\varphi\sin^2\psi+\cos^2\psi)=a^2$$

$$\Rightarrow EG-F^2=a^4\cos^2\psi\Rightarrow\sqrt{EG-F^2}=a^2|\cos\psi|=a^2\cos\psi$$

$$m=\iint_\Delta\sqrt{EG-F^2}\,d\varphi\,d\psi=\int_0^{2\pi}d\varphi\int_{\arcsin\frac{h}{a}}^{\frac{\pi}{2}}a^2\cos\psi\,d\psi=2\pi a^2[\sin\psi]_{\arcsin\frac{h}{a}}^{\frac{\pi}{2}}=2\pi a^2\left(1-\frac{h}{a}\right)$$

$$z_T=\frac{1}{m}\iint_\Delta\underbrace{a\sin\psi}_{z}\sqrt{EG-F^2}\,d\varphi\,d\psi=\frac{\cancel{a}}{2\pi\cancel{a}(a-h)}\int_0^{2\pi}d\varphi\int_{\arcsin\frac{h}{a}}^{\frac{\pi}{2}}\sin\psi\,a^2\cos\psi\,d\psi$$

$$=\frac{\cancel{2\pi}a^2}{\cancel{2\pi}(a-h)}\left[\frac{\sin^2\psi}{2}\right]_{\arcsin\frac{h}{a}}^{\frac{\pi}{2}}=\frac{a^2}{2(a-h)}\left[1-\left(\sin\left(\arcsin\frac{h}{a}\right)\right)^2\right]=$$

$$=\frac{a^2}{2(a-h)}\left[1-\frac{h^2}{a^2}\right]=\frac{\cancel{a^2}(a^2-h^2)}{2(a-h)\cancel{a^2}}=\frac{(a-h)(a+h)}{2(a-h)}=\frac{a+h}{2}\Rightarrow T\left(0,0,\frac{a+h}{2}\right)$$