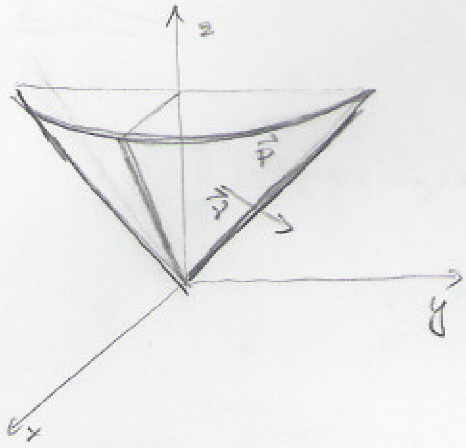
IZRAČUNAJ INTEGRAL $I = \iint_{\vec{P}} (x^2+y^2+z^2)\,dy\,dz$ PO TISTI STRANI STOŽCA $z^2 = x^2+y^2$, $0 \le z \le 1$, $x \ge 0$, KI JO VIDIMO S POZITIVNEGA DELA ABSCISNE OSI



$\vec{a} = (x^2+y^2+z^2, 0, 0)$

PARAMETRIZACIJA PLOSKVE S:

$\vec{f}(r,\varphi) = (r\cos\varphi, r\sin\varphi, r)$

$\Delta: r \in [0,1]$, $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\left.\begin{aligned}\vec{f}_r &= (\cos\varphi, \sin\varphi, 1)\\ \vec{f}_\varphi &= (-r\sin\varphi, r\cos\varphi, 0)\end{aligned}\right\} \vec{f}_r \times \vec{f}_\varphi = (-r\cos\varphi, -r\sin\varphi, r) \Rightarrow$$

$(-r\cos\varphi, -r\sin\varphi, r)$ IN $\vec{\nu}$ NISTA USKLAJENA $\Rightarrow$ PREDZNAK $-$

$$I = \iint_{\vec{S}} \vec{a}\,d\vec{S} = -\iint_{\Delta} [\vec{a}\circ\vec{f}, \vec{f}_r, \vec{f}_\varphi]\,dr\,d\varphi =$$

$$(\vec{a}\circ\vec{f})(r,\varphi) = \vec{a}(r\cos\varphi, r\sin\varphi, r) = (r^2\cos^2\varphi + r^2\sin^2\varphi + r^2, 0, 0)$$
$$= (2r^2, 0, 0)$$

$$= -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^1 (2r^2,0,0)\cdot(-r\cos\varphi, r\sin\varphi, r)\,dr = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^1 -2r^3\cos\varphi\,d\varphi$$

$$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi\,d\varphi \int_0^1 r^3\,dr = \cancel{4}\int_0^{\frac{\pi}{2}} \cos\varphi\,d\varphi \left[\frac{r^4}{\cancel{4}}\right]_0^1 = [\sin\varphi]_0^{\frac{\pi}{2}} = 1$$