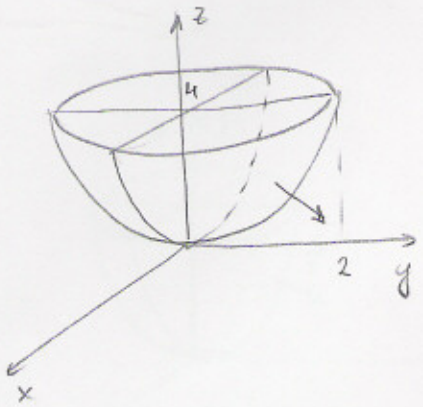
DN1

IZRAČUNAJ $\iint_{\vec{S}} x\,dy\,dz + z\,dx\,dy$ PO SPODNJI STRANI PLOSKVE

$S = \{(x,y,z) \in \mathbb{R}^3;\ 0 \le z \le 4,\ z = x^2+y^2\}$



$\vec{F}(x,y,z) = (x,0,z)$

$\vec{F} = (P,Q,R)$

$P = x$, $Q = 0$, $R = z$

$z(x,y) = x^2+y^2$

$p = z_x = 2x$

$q = z_y = 2y$

$\Delta = \{(x,y) \in \mathbb{R}^2;\ x^2+y^2 \le 4\}$

$$\iint_{\vec{P}} x\,dy\,dz + z\,dx\,dy = -\iint_{\Delta}(-pP - qQ + R)\,dx\,dy$$

$$= -\iint_{\Delta}(-2x\cdot x - 2y\cdot 0 + z)\,dx\,dy = -\iint_{\Delta}(-2x^2 + x^2+y^2)\,dx\,dy$$

$$= \iint_{\Delta}(x^2-y^2)\,dx\,dy = \int_0^{2\pi} d\varphi \int_0^2 (r^2\cos^2\varphi - r^2\sin^2\varphi)\,r\,dr$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$\varphi = r$

$$= \int_0^{2\pi}(\cos^2\varphi - \sin^2\varphi)\,d\varphi \int_0^2 r^3\,dr = 0$$

$$= \int_0^{2\pi} \cos 2\varphi\,d\varphi \left[\frac{r^4}{4}\right]_0^2 =$$

$$= 4\left[\frac{\sin 2\varphi}{2}\right]_0^{2\pi} = \underline{\underline{0}}$$