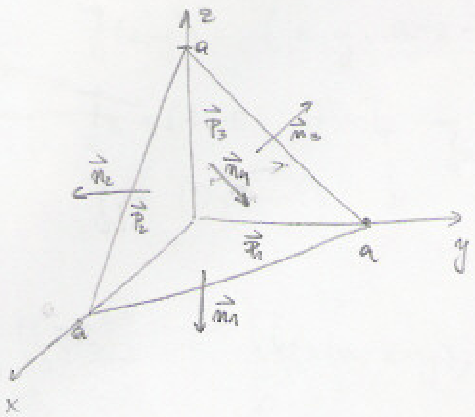
DN!

IZRAČUNAJ PRETOK POLJA $\vec F = (xz, xy, yz)$, KJER JE $\vec P$ ZUNANJA STRAN PIRAMIDE, KI JO DOLOČAJO $x=0, y=0, z=0$ IN $x+y+z=a$



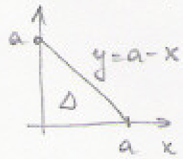
(a) DIREKTNO

$$\iint_{\vec P} \vec F \, d\vec S = \iint_{\vec P_1} \vec F \, d\vec S + \iint_{\vec P_2} \vec F \, d\vec S + \iint_{\vec P_3} \vec F \, d\vec S + \iint_{\vec P_4} \vec F \, d\vec S$$

$$= 0 + 0 + 0 + \frac{a^4}{8}$$

$$\boxed{\frac{a^4}{8}}$$

$x+y=a \Rightarrow y=a-x$



$\vec P_1$: $\vec m_1 = (0,0,-1)$

$\vec f(x,y) = (x,y,0)$ $\qquad x \in [0,a], \; y \in [0, a-x]$

$$\left.\begin{array}{l} \vec f_x(x,y) = (1,0,0) \\ \vec f_y(x,y) = (0,1,0) \end{array}\right\} \vec f_x \times \vec f_y = (0,0,1) = -\vec m_1$$

NASPROTNI ORIENTACIJI

$$\iint_{\vec P_1} \vec F \, d\vec S = - \iint_{\Delta} \underbrace{(0, xy, 0)}_{\vec F(\vec f(x,y))} (0,0,1) \, dx\,dy = \underline{0}$$

$\vec P_2$: $\vec m_2 = (0,-1,0)$

$\vec f(x,z) = (x,0,z)$ $\qquad x \in [0,a], \; z \in [0, a-z]$

$$\left.\begin{array}{l} \vec f_x = (1,0,0) \\ \vec f_z = (0,0,1) \end{array}\right\} \vec f_x \times \vec f_z = (0,-1,0) = +\vec m_2$$

$$\iint_{\vec P_2} \vec F \, d\vec S = + \iint_{\Delta} (xz, 0, 0)(0,-1,0) \, dx\,dz = \underline{0}$$

$\vec P_3$: $\vec m_3 = (-1,0,0)$

$\vec f(y,z) = (0,y,z)$ $\qquad y \in [0,a], \; z \in [0, a-y]$

$$\left.\begin{array}{l} \vec f_y = (0,1,0) \\ \vec f_z = (0,0,1) \end{array}\right\} \vec f_y \times \vec f_z = (1,0,0) = -\vec m_3$$

$$\iint\limits_{\vec{P}_3} \vec{F}\,\vec{dS} = -\iint\limits_{\Delta} (0,0,yz)(1,0,0)\,dy\,dz = \underline{\underline{0}}$$

$\vec{P}_4$:
$$\vec{g}(x,y) = (x, y, a-x-y)$$
$$\left.\begin{array}{l}\vec{g}_x = (1,0,-1)\\ \vec{g}_y = (0,1,-1)\end{array}\right\} \quad \vec{g}_x \times \vec{g}_y = (1,1,1) = \sqrt{3}\,\vec{m}_4 \text{ (KER KAŽE NAVZGOR)}$$

$$\iint\limits_{\vec{P}_4} \vec{F}\,\vec{dS} = \iint\limits_{\Delta} (x(a-x-y),\ xy,\ y(a-x-y))(1,1,1)\,dx\,dy$$

$$= \int_0^a dx \int_0^{a-x} [ax - x^2 - \cancel{xy} + \cancel{xy} + ay - xy - y^2]\,dy$$

$$= \int_0^a dx \left[axy - x^2y + \underbrace{a\frac{y^2}{2} - x\frac{y^2}{2}} - \frac{y^3}{3}\right]_0^{a-x}$$

$$= \int_0^a ax(a-x) - x^2(a-x) + \frac{(a-x)^3}{2} - \frac{(a-x)^3}{3}\Big]\,dx$$

$$= \int_0^a \left(x(a-x)^2 + \frac{(a-x)^3}{6}\right)dx = \frac{1}{6}\int_0^a (a-x)^2(6x + a - x)\,dx =$$

$$= \frac{1}{6}\int_0^a (a-x)^2(5x+a)\,dx = \frac{1}{6}\int_0^a (a^2 - 2ax + x^2)(5x+a)\,dx$$

$$= \frac{1}{6}\int_0^a (5a^2x - 10ax^2 + 5x^3 + a^3 - 2a^2x + x^2a)\,dx$$

$$= \frac{1}{6}\int_0^a (3a^2x - 9ax^2 + 5x^3 + a^3)\,dx$$

$$= \frac{1}{6}\left[3a^2\frac{x^2}{2} - \frac{9ax^3}{3} + \frac{5x^4}{4} + a^3x\right]_0^a =$$

$$= \frac{1}{6}\left[\frac{3a^4}{2} - \frac{\overset{3}{\cancel{9}}a^4}{\cancel{3}} + \frac{5a^4}{4} + a^4\right] = \frac{a^4}{6}\left[\frac{3}{2} - 3 + \frac{5}{4} + 1\right]$$

$$= \frac{a^4}{24}[6 - 12 + 5 + 4] = \frac{a^4 \cdot 3}{24} = \boxed{\frac{a^4}{8}}$$

(B) Z GAUSSOVIM IZREKOM

$$\iint_{\vec{P}=\partial G} \vec{F}\,d\vec{S} = \iiint_G \operatorname{div}\vec{F}\,dV =$$

$$\vec{F}=(xz, xy, yz) \Rightarrow \operatorname{div}\vec{F} = z+x+y$$

$$= \int_0^a dx \int_0^{a-x} dy \int_0^{a-x-y} (x+y+z)\,dz = \int_0^a dx \int_0^{a-x} \left[(x+y)z + \frac{z^2}{2} \right]_0^{a-x-y} dy$$

$$= \int_0^a dx \int_0^{a-x} \left[(x+y)(a-x-y) + \frac{(a-x-y)^2}{2} \right] dy$$

$$= \int_0^a dx \int_0^{a-x} \frac{1}{2}(a-x-y)(2x+2y+a-x-y)\,dy$$

$$= \frac{1}{2}\int_0^a dx \int_0^{a-x} (a-x-y)(a+x+y)\,dy = \frac{1}{2}\int_0^a dx \int_0^{a-x} (a^2-(x+y)^2)\,dy$$

$$= \frac{1}{2}\int_0^a dx \left[a^2 y - \frac{(x+y)^3}{3} \right]_0^{a-x} = \frac{1}{2}\int_0^a \left(a^2(a-x) - \frac{(x+a-x)^3}{3} + \frac{x^3}{3} \right) dx$$

$$= \frac{1}{2}\int_0^a \left[a^3 - a^2 x - \frac{a^3}{3} + \frac{x^3}{3} \right] dx = \frac{1}{6}\int_0^a \left[2a^3 - 3a^2 x + x^3 \right] dx$$

$$= \frac{1}{6}\left[2a^3 x - \frac{3a^2x^2}{2} + \frac{x^4}{4} \right]_0^a = \frac{1}{6}\left[2a^4 - \frac{3a^4}{2} + \frac{a^4}{4} \right]$$

$$= \frac{a^4}{24}\left[8-6+1 \right] = \frac{3a^4}{24} = \boxed{\frac{a^4}{8}}$$