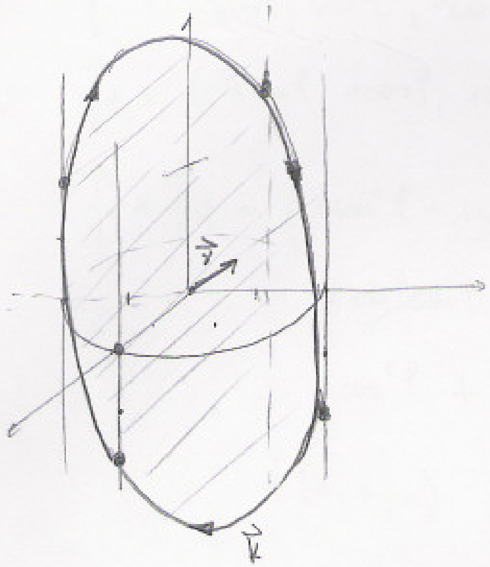
[illegible]

NAJ BO KRIVULJA K DANA KOT PRESEK PLOSKEV $x^2+y^2=4$ IN $x+y+z=0$, ORIENTIRANA V SMERI ROTACIJE URINEGA KAZALCA, ČE JO GLEDAMO IZ TOČKE $(0,0,100)$. SKICIRAJTE $\vec{K}$ IN IZRAČUNAJTE

$$\oint_{\vec{K}} yz\,dx + xz\,dy + x^2\,dz$$



x	y	$z=-x-y$
2	0	-2
0	2	-2
-2	0	2
0	-2	2

$$\vec{U}(x,y,z)=(yz,\ xz,\ x^2)$$

(a) DIREKTNO

$$\vec{K}:\ \vec{r}(t)=(2\cos t,\ 2\sin t,\ -2\cos t-2\sin t) \qquad t\in[0,2\pi]$$

$$\dot{\vec{r}}(t)=(-2\sin t,\ 2\cos t,\ 2\sin t-2\cos t)$$

$$\oint_{\vec{K}}\vec{U}\,d\vec{r} = -\int_0^{2\pi}\left(2\sin t(-2\cos t-2\sin t),\ 2\cos t(-2\cos t-2\sin t),\ 4\cos^2 t\right).$$

KER STA ORIENTACIJA
$\vec{K}$ IN TISTA, KI JO
INDUCIRA PARAMETRIZACIJA
NASPROTNI

$$\cdot(-2\sin t,\ 2\cos t,\ 2\sin t-2\cos t)\,dt$$

$$= -8\int_0^{2\pi}\left(\sin^2 t\cos t + \sin^3 t - \cos^3 t - \cos^2 t\sin t - \cos^3 t\right)dt$$

$$= -8\Big[\underbrace{\int_0^{2\pi}\sin^2 t\cos t\,dt}_{0} + \underbrace{\int_0^{2\pi}\sin^3 t\,dt}_{0} - 2\underbrace{\int_0^{2\pi}\cos^3 t\,dt}_{0} - \underbrace{\int_0^{2\pi}\cos^2 t\sin t\,dt}_{0}\Big] = \underline{\underline{0}}$$

(B) Z UPORABO STOKESOVE FORMULE

$$\vec{r}(r,\varphi) = (r\cos\varphi,\ r\sin\varphi,\ -r\cos\varphi - r\sin\varphi) \qquad \varphi\in[0,2\pi],\ r\in[0,2]$$

$$\vec{r}_r = (\cos\varphi, \sin\varphi, -\cos\varphi - \sin\varphi)$$

$$\vec{r}_\varphi = (-r\sin\varphi, r\cos\varphi, r\sin\varphi - r\cos\varphi)$$

$$\vec{r}_r \times \vec{r}_\varphi = \big(r\sin^2\varphi - r\cancel{\sin\varphi\cos\varphi} + r\cos^2\varphi + r\cancel{\sin\varphi\cos\varphi},$$
$$r\cancel{\sin\varphi\cos\varphi} + r\sin^2\varphi - r\cancel{\sin\varphi\cos\varphi} + r\sin^2\varphi,$$
$$r\cos^2\varphi + r\sin^2\varphi\big)$$

$$= (r,r,r) \quad \Rightarrow \quad \vec{r}_r \times \vec{r}_\varphi \text{ IN } \vec{v} \text{ KAŽETA V ISTO SMER}$$

$$\oint_{\vec{K}} \vec{U}\,d\vec{r} = \int_{\vec{P}} \operatorname{rot}\vec{U}\,d\vec{S} = \int_0^{2\pi} d\varphi \int_0^2 (-r\cos\varphi,\ r\sin\varphi - 2r\cos\varphi,\ 0)(r,r,r)\,dr$$

$$\operatorname{rot}\vec{U} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & x^2 \end{vmatrix} = (-x,\ y-2x,\ z-z) = (-x, y-2x, 0)$$

$$= \int_0^{2\pi} d\varphi \int_0^2 (-r^2\cos\varphi + r^2\sin\varphi - 2r^2\cos\varphi)\,dr = \underbrace{\int_0^{2\pi}(\sin\varphi - 2\cos\varphi)\,d\varphi}_{0}\int_0^2 r^2\,dr = \underline{\underline{0}}$$

EKSPLICITNO :

$z(x,y) = -x-y$

$p=-1$

$q=-1$

$D=\{(x,y)\in\mathbb{R}^2 \,;\, x^2+y^2\le 4\}$

$$\oint_{\vec{K}} \vec{U}\,d\vec{r} = \int_{\vec{P}} \mathrm{rot}\,\vec{U}\,\vec{dS} = \underset{\uparrow}{+}\iint_D (-x,\, y-2x,\, 0)\underbrace{(1,1,1)}_{(-p,-q,1)}\,dx\,dy$$

INTEGRIRAMO
PO ZGORNJI STRANI

$$= \iint_D (-x+y-2x)\,dx\,dy = \iint_D (y-3x)\,dx\,dy$$

UVEDEMO POLARNE KOORDINATE:

$x = r\cos\varphi$

$y = r\sin\varphi$

$J = r$

$$= \int_0^{2\pi} d\varphi \int_0^2 (r\sin\varphi - 3r\cos\varphi)\,\underset{\substack{\uparrow\\|J|}}{r}\,dr = \underbrace{\int_0^{2\pi}(\sin\varphi - 2\cos\varphi)\,d\varphi}_{0}\int_0^2 r^2\,dr = \underline{0}|$$