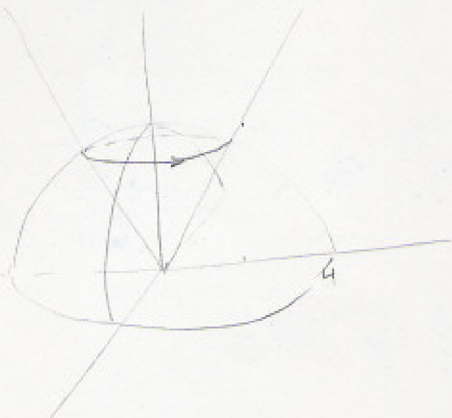
11se P.N.

Izračunaj $\oint_{\vec K} yz^2\,dx + xy^2\,dy + zx^2\,dz$ vzdolž K, ki je presek

$x^2+y^2+z^2=10$ in $z=2\sqrt{x^2+y^2}$, kjer je orientacija nasprotna

urinemu kazalcu, če gledamo z vrha osi z.



$z^2 = 10 - x^2 - y^2 = 4(x^2+y^2)$

$10 = 5(x^2+y^2)$

$2 = x^2+y^2$

$z = 2\sqrt2$

direktno

$x = \sqrt2\cos t$

$y = \sqrt2 \sin t$

$z = 2\sqrt2$

$$\oint_{\vec K}\vec v\,d\vec r = +\int_0^{2\pi}\left(8\sqrt2\sin t,\ 2\sqrt2\sin^2 t\cos t,\ 4\sqrt2\cos^2 t\right)\left(-\sqrt2\sin t,\ \sqrt2\cos t,\ 0\right)dt$$

$$= \int_0^{2\pi}\left(-16\sin^2 t + 4\sin^2 t\cos^2 t\right)dt = -\frac{64}{2}B\left(\tfrac32,\tfrac12\right) + \frac{16}{2}B\left(\tfrac32,\tfrac32\right)$$

$$= +32\cdot\frac{\frac12\sqrt\pi\sqrt\pi}{1} + 8\,\frac{\frac12\sqrt\pi\ \frac12\sqrt\pi}{2} = -16\pi + \pi = -15\pi$$

Stokes:

$$\operatorname{rot}\vec v = \begin{vmatrix}\vec i & \vec j & \vec k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ yz^2 & xy^2 & zx^2\end{vmatrix} = \left(0,\ 2yz-2xz,\ y^2-z^2\right)$$

$$\oint_{\vec K}\vec v\,d\vec r = +\iint_{\vec P}\left(0,2yz-2xz,y^2-z^2\right)d\vec S = +\iint_D\left(0,2yz-2xz,y^2-z^2\right)(0,0,1)\,dx\,dy$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$= \int_0^{2\pi}d\varphi\int_0^{\sqrt2}\left(r^2\sin^2\varphi - 8\right)r\,dr = \int_0^{2\pi}\sin^2\varphi\left[\frac{r^4}{4}\right]_0^{\sqrt2}d\varphi - 8\cdot 2\pi\left[\frac{r^2}{2}\right]_0^{\sqrt2}$$

$$= \int_0^{2\pi}\sin^2\varphi\,d\varphi - 16\pi = 4\cdot\frac12 B\left(\tfrac32,\tfrac12\right) - 16\pi = 2\cdot\frac{\frac12\sqrt\pi\sqrt\pi}{1} - 16\pi =$$

$$= \underline{\underline{-15\pi}}$$