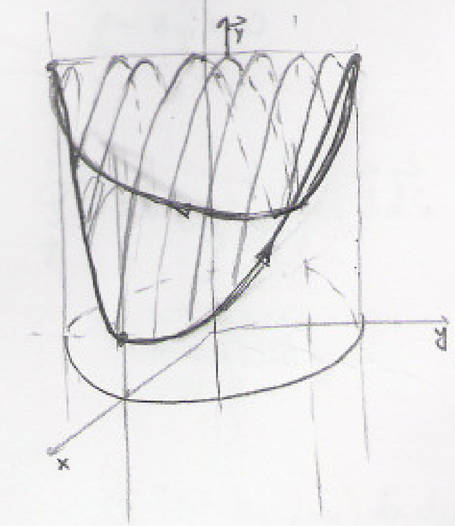
NAJ BO K PRESEK PLOSKEV $x^2+y^2=4$ IN $z=5-x^2$, ORIENTIRANA V SMERI URINEGA KAZALCA, ČE JO GLEDAMO IZ IZHODIŠČA. IZRAČUNAJTE $\oint \vec{U}\,d\vec{r}$, ČE JE $\vec{U}(x,y,z)=(xy^2, z, xy)$



x	y	z
2	0	1
0	2	5
-2	0	1
0	-2	5

(a) DIREKTNO $\vec{K}: \vec{r}(t)=(2\cos t, 2\sin t, 5-4\cos^2 t) \quad t\in[0,2\pi]$

$\dot{\vec{r}}(t)=(-2\sin t, 2\cos t, +8\cos t\sin t)$

$$\oint_{\vec{K}} \vec{U}\,d\vec{r} = +\int_0^{2\pi} (2\cos t\, 4\sin^2 t,\ 5-4\cos^2 t,\ 2\cos t\, 2\sin t)(-2\sin t, 2\cos t, 8\cos t\sin t)\,dt$$

$$=\int_0^{2\pi}(-16\cos t\sin^3 t + 10\cos t - 8\cos^3 t + 32\cos^2 t\sin^2 t)\,dt$$

$$=-16\underbrace{\int_0^{2\pi}\cos t\sin^3 t\,dt}_{0} + 10\underbrace{\int_0^{2\pi}\cos t\,dt}_{0} - 8\underbrace{\int_0^{2\pi}\cos^3 t\,dt}_{0} + 32\int_0^{2\pi}\sin^2 t\cos^2 t\,dt$$

$$=32\cdot 4\cdot \frac{1}{2} B\left(\frac{3}{2},\frac{3}{2}\right) = \frac{32\cdot \cancel{2}\,\frac{1}{2}\sqrt{\pi}\,\frac{1}{2}\sqrt{\pi}}{\cancel{2}} = \underline{\underline{8\pi}}$$

(b) S STOKESOVIM IZREKOM

$$\operatorname{rot}\vec{U} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & z & xy \end{vmatrix} = (x-1, 0-y, 0-2xy)$$

$$=(x-1, -y, -2xy)$$

EKSPLICITNO

$z = 5 - x^2$ $\quad P = \{(x,y,z) \in \mathbb{R}^3 ;\ x^2+y^2 \leq 4,\ z = 5-x^2\}$

$$\left.\begin{array}{l} p = z_x = -2x \\ q = z_y = 0 \end{array}\right\} \quad \vec{n} = (-p, -q, 1) = (2x, 0, 1)$$

$$\oint_K \vec{U}\, d\vec{r} = +\iint_{\vec{P}} \operatorname{rot}\vec{U}\, d\vec{S} = \iint_D (x-1, -y, -2xy)(2x, 0, 1)\, dx\, dy$$

INTEGRIRAMO PO ZGORNJI STRANI

$D = \{(x,y) \in \mathbb{R}^2 ;\ x^2+y^2 \leq 4\}$

$$= \iint_D (2x^2 - 2x - 2xy)\, dx\, dy = \int_0^{2\pi} d\varphi \int_0^2 (2r^2\cos^2\varphi - 2r\cos\varphi - 2r^2\cos\varphi\sin\varphi)\, r\, dr \quad (*)$$

UVEDEMO POLARNE KOORDINATE $\quad x = r\cos\varphi,\ y = r\sin\varphi,\ J = r$

$$= 2\int_0^{2\pi} \cos^2\varphi\, d\varphi \int_0^2 r^3 dr - 2\underbrace{\int_0^{2\pi} \cos\varphi\, d\varphi}_{0} \int_0^2 r^2 dr - 2\underbrace{\int_0^{2\pi} \cos\varphi\sin\varphi\, d\varphi}_{0} \int_0^2 r^3 dr$$

$$= \cancel{2} \cdot 4 \cdot \tfrac{1}{\cancel{2}} B\left(\tfrac{1}{2}, \tfrac{3}{2}\right) \left[\tfrac{r^4}{4}\right]_0^2 = \frac{\pi \cdot \tfrac{1}{2}\pi \ldots 16 \cdot 8}{1} = \underline{\underline{8\pi}}$$

PARAMETRIČNO: $P: \vec{f}(r,\varphi) = (r\cos\varphi, r\sin\varphi, 5 - r^2\cos^2\varphi) \quad r \in [0,2],\ \varphi \in [0, 2\pi]$

$$\left.\begin{array}{l} \vec{f}_r = (\cos\varphi, \sin\varphi, -2r\cos^2\varphi) \\ \vec{f}_\varphi = (-r\sin\varphi, r\cos\varphi, +2r^2\cos\varphi\sin\varphi) \end{array}\right\} \Rightarrow$$

$$\vec{f}_r \times \vec{f}_\varphi = (2r^2\cos\varphi\sin^2\varphi + 2r^2\cos^3\varphi,\ 2r^2\sin\varphi\cos^2\varphi - 2r^2\sin\varphi\cos^2\varphi,\ r\cos^2\varphi + r\sin^2\varphi)$$

$= (2r^2\cos\varphi, 0, r) \Rightarrow \vec{f}_r \times \vec{f}_\varphi$ IN $\vec{n}$ KAŽETA V ISTO SMER

$$\oint_K \vec{U}\, d\vec{r} = +\iint_{\vec{P}} \operatorname{rot}\vec{U}\, d\vec{S} = \int_0^{2\pi} d\varphi \int_0^2 (r\cos\varphi - 1, -r\sin\varphi, -2r^2\cos\varphi\sin\varphi)(2r^2\cos\varphi, 0, r)\, dr$$

$$= \int_0^{2\pi} d\varphi \int_0^2 (2r^3\cos^2\varphi - 2r^2\cos\varphi - 2r^3\cos\varphi\sin\varphi)\, dr \overset{\text{GLEJ } (*)}{=} \underline{8\pi}$$