MSK

DN.

ZA VEKTORSKO POLJE

$$\vec{U}(x,y,z) = (ze^x, z^2, 2yz + e^x)$$

(a) PREVERI, DA JE POTENCIALNO NA $\mathbb{R}^3$

(b) IZRAČUNAJ NJEGOV SKALARNI POTENCIAL

(c) IZRAČUNAJ

$$I = \int_{K:A}^{B} \vec{U}\,d\vec{r},$$

ČE JE $K: \vec{r}(t) = (t^2, 1-t, t^4 - 2t + 1)$, $t \in [0,1]$

(a) KER JE $\mathbb{R}^3$ ENOSTAVNO POVEZANO OBMOČJE, JE DOVOLJ, ČE PREVERIMO, DA JE $\operatorname{rot}\vec{U} = \vec{0}$

$$\operatorname{rot}\vec{U} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ze^x & z^2 & 2yz+e^x \end{vmatrix} = (2z - 2z, e^x - e^x, 0-0) = \vec{0}$$

(b) REŠUJEMO SISTEM:

$u_x \overset{*}{=} ze^x$

$u_y \overset{**}{=} z^2$

$u_z \overset{***}{=} 2yz + e^x$

$u_x \overset{*}{=} ze^x \Rightarrow u \overset{(1)}{=} ze^x + \varphi(y,z)$

$\Rightarrow \left.\begin{array}{l} u_y = \varphi_y(y,z) \\ u_y \overset{**}{=} z^2 \end{array}\right\} \Rightarrow$

$\Rightarrow \varphi_y(y,z) = z^2$

$\Rightarrow \varphi(y,z) = yz^2 + \psi(z)$

$\overset{(1)}{\Rightarrow} u \overset{(2)}{=} ze^x + yz^2 + \psi(z)$

$\Rightarrow \left.\begin{array}{l} u_z = e^x + 2yz + \psi'(z) \\ u_z \overset{***}{=} 2yz + e^x \end{array}\right\} \Rightarrow \psi'(z) = 0$

$\Rightarrow \psi(z) = C \overset{(2)}{\Longrightarrow} u = ze^x + yz^2 + C$

$u(x,y,z) = ze^x + yz^2 + c$

(c) $\int_{K:A}^{B} \vec{U}\, d\vec{r} = \int_{K:A}^{B} du = u(B) - u(A)$

$= u(1,0,0) - u(0,1,1)$

$= (0e^1 + 0 \cdot 0^2 + c) - (1e^0 + 1 \cdot 1^2 + c)$

$= c - 1 - 1 - c$

$= \underline{-2}$