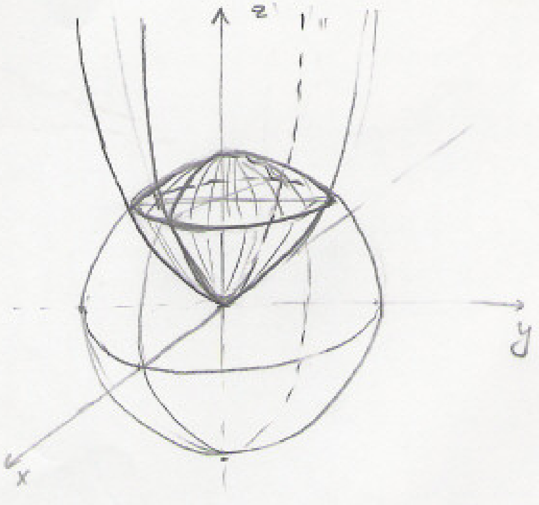
DN 6

POIŠČI VOLUMEN TELESA OMEJENEGA S SFERO $x^2+y^2+z^2=4$ IN PARABOLOIDOM $x^2+y^2=3z$



PRESEK SFERE IN PARABOLOIDA

$$\left.\begin{array}{l} x^2+y^2+z^2=4 \\ x^2+y^2=3z \end{array}\right\} \quad \begin{array}{l} z^2+3z-4=0 \\ (z+4)(z-1)=0 \end{array}$$

$z_1 = 1$

$z_2 = -4$ //

$z_1 = 1: \quad x^2+y^2=3$

KROŽNICA S POLMEROM $\sqrt{3}$

VPELJEMO CILINDRIČNE KOORDINATE:

$$\left.\begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{array}\right\} \Rightarrow \begin{array}{l} x^2+y^2+z^2=4 \\ z^2 = 4-r^2 \\ z = \pm\sqrt{4-r^2} \end{array}$$

$J = r$

$$\begin{array}{l} x^2+y^2=3z \\ r^2 = 3z \\ z = \frac{r^2}{3} \end{array}$$

$$V = \int_0^{2\pi} d\varphi \int_0^{\sqrt{3}} r\,dr \int_{\frac{r^2}{3}}^{\sqrt{4-r^2}} dz =$$

$$= 2\pi \int_0^{\sqrt{3}} r\,[z]_{\frac{r^2}{3}}^{\sqrt{4-r^2}}\,dr = 2\pi \int_0^{\sqrt{3}} \left[r\sqrt{4-r^2} - \frac{r^3}{3}\right] dr =$$

$4-r^2 = t^2$

$-\cancel{2}r\,dr = \cancel{2}t\,dt$

$$= 2\pi \left[\int_2^1 |t|(-t)\,dt - \frac{r^4}{12}\Big|_0^{\sqrt{3}} \right] = 2\pi \left[\int_1^2 t^2\,dt - \frac{9}{12} \right]$$

$$= 2\pi \left[\frac{t^3}{3}\Big|_1^2 - \frac{3}{4} \right] = 2\pi \left[\frac{7}{3} - \frac{3}{4} \right] = \frac{\cancel{2}\pi\,(28-9)}{\cancel{12}\,6} = \boxed{\frac{19\pi}{6}}$$