POIŠČI ORTOGONALNE TRAJEKTORIJE DRUŽINE $y = D(x^2+y^2)$ DN 63

(a) POIŠČIMO D.E. DANE DRUŽINE

$$\left.\begin{array}{l} y' = D(2x+2yy') \\ y = D(x^2+y^2) \Rightarrow D = \frac{y}{x^2+y^2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow y' = \frac{y}{x^2+y^2}(2x+2yy')$$

$$y'(x^2+y^2) = 2xy + 2y^2y'$$

$$x^2y' + y^2y' = 2xy + 2y^2y'$$

$$x^2y' - y^2y' = 2xy$$

$$y' = \frac{2xy}{x^2-y^2}$$

(b) D.E. ORT. TRAJEKTORIJ : $y' \rightsquigarrow -\frac{1}{y'}$

$$-\frac{1}{y'} = \frac{2xy}{x^2-y^2}$$

$$y' = \frac{y^2-x^2}{2xy} = \frac{\frac{y^2}{x^2}-1}{2\frac{y}{x}} = \frac{\left(\frac{y}{x}\right)^2-1}{2\frac{y}{x}}$$

(D.E. S HOMOGENO DESNO STRANJO)

(c) $y' = \dfrac{\left(\frac{y}{x}\right)^2-1}{2\left(\frac{y}{x}\right)}$ $\qquad \frac{y}{x} = u \Rightarrow y = ux \Rightarrow y' = u'x + u$

$$u'x + u = \frac{u^2-1}{2u}$$

$$u'x = \frac{u^2-1-2u^2}{2u} = \frac{-1-u^2}{2u}$$

$$\frac{2u}{1+u^2}\,du = -\frac{dx}{x}$$

$$\ln|1+u^2| = -\ln|x| + \ln C \quad , C>0$$

$$|1+u^2| = \frac{C}{|x|}$$

$$1+u^2 = \frac{D}{x} \qquad D \neq 0$$

$$1 + \frac{y^2}{x^2} = \frac{D}{x}$$

$$\boxed{x^2+y^2 = Dx}$$ ENAČBE ORTOG. TRAJEKTORIJ