

Penyelesaian:

$$y'' + y = x^2 - x + 1$$

$$y(0) = -1, \quad y'(0) = 2$$

homogen: $y'' + y = 0$

$$\lambda^2 + 1 = 0 \rightarrow \lambda = \pm i$$

solusi homogen: $y_h = C_1 \cos x + C_2 \sin x$

non-homogen: $b(x) = e^{ax} P_m(x)$

$$= x^2 - x + 1$$

$$a=0, m=2$$

misalkan: $x^n e^{ax} Q_m(x) = Ax^2 + Bx + C$

$$y = Ax^2 + Bx + C$$

$$y' = 2Ax + B$$

$$y'' = 2A$$

$$y'' + y = x^2 - x + 1$$

$$2A + Ax^2 + Bx + C = x^2 - x + 1$$

$$A = 1$$

$$B = -1$$

$$2A + C = 1 \Rightarrow C = 1 - 2A = -1$$

solusi non-homogen: $y(x) = C_1 \cos x + C_2 \sin x + x^2 - x - 1$

$$y(0) = C_1 - 1 = -1 \Rightarrow C_1 = 0$$

$$y' = -C_2 \sin x + C_2 \cos x + 2x - 1$$

$$y'(0) = C_2 + 1 = 2 \Rightarrow C_2 = 1$$

$$y(x) = \sin x + x^2 - x - 1$$