DN

66.

$y'' - 4y' + 4y = xe^{2x}$ $\quad y(0)=1,\ y'(0)=4$

homogena enačba $\quad y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$(\lambda-2)^2 = 0 \quad \lambda_{1,2} = 2$

$y_H = C_1 e^{2x} + C_2 x e^{2x}$

nek. enačba: $\quad b(x) = xe^{2x} \Rightarrow a=2,\ m=1$

ker je 2 dvojni koren kar. polinoma:

$\tilde{y} = x^2 e^{2x}(Ax+B) = (Ax^3+Bx^2)e^{2x}$

$\tilde{y} = (Ax^3+Bx^2)e^{2x}$

$\tilde{y}' = (3Ax^2+2Bx + 2Ax^3+2Bx^2)e^{2x}$

$= (2Ax^3 + (3A+2B)x^2 + 2Bx)e^{2x}$

$\tilde{y}'' = (6Ax^2+(6A+4B)x+2B+4Ax^3+(6A+4B)x^2+4Bx)e^{2x}$

$= (4Ax^3 + (12A+4B)x^2 + (6A+8B)x + 2B)e^{2x}$

$\tilde{y}'' - 4y' + 4y = (4Ax^3 + (12A+4B)x^2 + (6A+8B)x + 2B$

$-8Ax^3 - (12A+8B)x^2 - 8Bx$

$+4Ax^3 + 4Bx^2)e^{2x} = (6Ax+2B)e^{2x} = xe^{2x} \Rightarrow$

$\Rightarrow 6A = 1 \Rightarrow A = \frac{1}{6}$

$2B = 0 \Rightarrow B = 0$

$y = C_1 e^{2x} + C_2 x e^{2x} + \frac{1}{6}x^3 e^{2x}$

$y(0) = C_1 = 1$

$y' = 2C_1 e^{2x} + C_2 e^{2x} + 2C_2 x e^{2x} + \frac{1}{2}x^2 e^{2x} + \frac{1}{3}x^5 e^{2x}$

$y'(0) = 2C_1 + C_2 = 4 \Rightarrow C_2 = 2$

$y = e^{2x} + 2xe^{2x} + \frac{1}{6}x^3 e^{2x}$