Rješenje

DN
67

(a) $y'' + y = 2\sin x \qquad y(0) = 3 \;,\; y'(0) = 1$

hom. rješ. $\quad y'' + y = 0$

$\lambda^2 + 1 = 0$

$\lambda = \pm i \quad \Rightarrow y_H = C_1 \cos x + C_2 \sin x$

$b(x) = e^{ax}\left[P_m(x)\cos bx + R_n(x)\sin bx\right]$

$= 2\sin x$

$a = 0 \qquad N = 0$

$b = 1$

ker je i rješ. karakt. jednadžbe je $\lambda = 1$

partikularno $\tilde{y} = x^k e^{ax}\left[Q_N(x)\cos bx + R_N(x)\sin bx\right]$

$= x \cdot e^{0x}\left[A\cos x + B\sin x\right] = x\left[A\cos x + B\sin x\right]$

$\tilde{y}' = A\cos x + B\sin x + Ax\sin x + Bx\cos x$

$\tilde{y}'' = -A\sin x + B\cos x - Ax\cos x - A\sin x + B\cos x - Bx\sin x$

$\tilde{y}'' + \tilde{y} = -Ax\cos x + Bx\sin x + 2B\cos x - 2A\sin x$

$+ Ax\cos x + Bx\sin x = 2B\cos x - 2A\sin x = 2\sin x \Rightarrow$

$\Rightarrow B = 0 \;,\; A = -1$

$y = C_1\cos x + C_2\sin x - x\cos x$

$y(0) = C_1 = 3$

$y'(x) = -C_1\sin x + C_2\cos x - \cos x + x\sin x$

$y'(0) = C_2 - 1 = 1 \Rightarrow C_2 = 2$

R: $y = 3\cos x + 2\sin x - x\cos x$