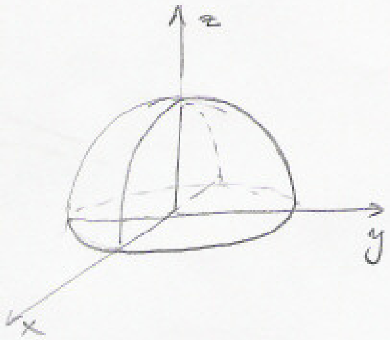
IZRAČUNAJ KOORDINATE TEŽIŠČA POLKROGLE $x^2+y^2+z^2 \le R^2,\ z \ge 0$, ČE JE GOSTOTA SORAZMERNA ODDALJENOSTI OD IZHODIŠČA



$$\rho(x,y,z) = C\sqrt{x^2+y^2+z^2}$$

ZARADI SIMETRIJE JE

$$x_T^* = y_T^* = 0$$

VPELJEMO SFERIČNE KOORDINATE

$$x = r\cos\psi\cos\varphi$$
$$y = r\cos\psi\sin\varphi$$
$$z = r\sin\psi$$
$$J = r^2\cos\psi$$

$$m(G) = \iiint_G \rho(x,y,z)\,dx\,dy\,dz =$$
$$= \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\psi \int_0^R \underbrace{Cr}_{\rho}\ \underbrace{r^2\cos\psi}_{J}\, dr$$
$$= C\cdot \cancel{2}\pi \int_0^{\frac{\pi}{2}} \cos\psi\, d\psi \left[\frac{r^4}{\cancel{4}2}\right]_0^R$$
$$= \frac{CR^4\pi}{2}\sin\psi\Big|_0^{\frac{\pi}{2}} = \boxed{\frac{CR^4\pi}{2}}$$

$$\iiint_G z\,\rho(x,y,z)\,dx\,dy\,dz = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\psi \int_0^R \underbrace{r\sin\psi}_{z}\ \underbrace{Cr}_{\rho}\ \underbrace{r^2\cos\psi}_{J}\, dr$$
$$= C\int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \sin\psi\cos\psi\, d\psi \left[\frac{r^5}{5}\right]_0^R = \cancel{2}\pi\frac{CR^5}{5}\cdot\frac{1}{\cancel{2}}B(1,1) = \frac{CR^5\pi\cdot 1\cdot 1}{5\cdot 1} = \frac{CR^5\pi}{5}$$

$$\Rightarrow\quad z_T^* = \frac{1}{m}\iiint_G x\,\rho(x,y,z)\,dx\,dy\,dz = \frac{C\cancel{R^5}\cancel{\pi}\cdot 2}{5\cdot \cancel{CR^4\pi}} = \frac{2R}{5}$$

$T\left(0,0,\frac{2R}{5}\right)$ ... TEŽIŠČE