DN1
9

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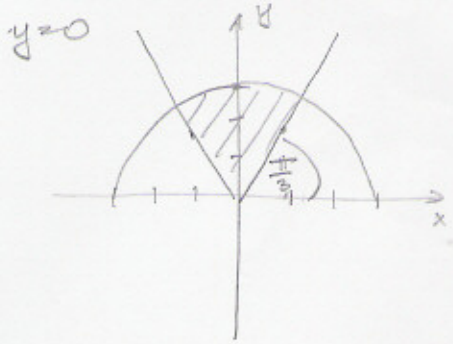
$$G = \{(x,y,z) \in \mathbb{R}^3;\ x^2+y^2+z^2 \le 9,\ z \ge \sqrt{3(x^2+y^2)}\}$$

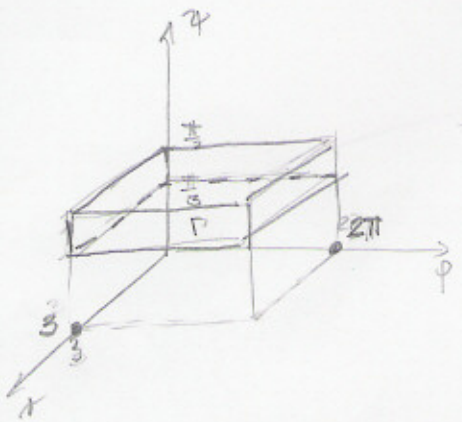


$x^2+z^2 \le 9$

$z \ge \sqrt{3}|x|$

$\operatorname{tg}\varphi = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\Rightarrow \varphi = \frac{\pi}{3}$



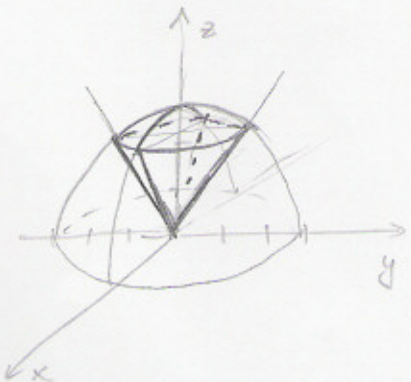
T

$x = r\cos\varphi\cos\psi$

$y = r\sin\varphi\cos\psi$

$z = r\sin\psi$

$y = r^2\cos\psi$



$$m = \iiint_G dx\,dy\,dz = \iiint_\Gamma r^2\cos\psi\,dr\,d\varphi\,d\psi = \int_0^{2\pi} d\varphi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos\psi\,d\psi \int_0^3 r^2\,dr$$

$$= 2\pi\left[\sin\psi\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}\left[\frac{r^3}{3}\right]_0^3 = 2\pi\left(1-\frac{\sqrt{3}}{2}\right)\frac{27}{3} = \frac{2\pi(2-\sqrt{3})\cdot 9}{2} = 9\pi(2-\sqrt{3})$$

- $x_T = y_T = 0$ ZARADI SIMETRIJE

$$z_T = \frac{1}{m}\iiint_G z\,dx\,dy\,dz = \frac{1}{m}\iiint_\Gamma r\sin\psi\, r^2\cos\psi\,dr\,d\varphi\,d\psi$$

$$= \frac{1}{m}\int_0^{2\pi} d\varphi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\psi\cos\psi\,d\psi \int_0^3 r^3\,dr = \frac{1}{m}\cdot 2\pi\left[\frac{\sin^2\psi}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}\left[\frac{r^4}{4}\right]_0^3$$

$$= \frac{2\pi\left(1-\frac{3}{4}\right)\cdot 2\cdot 9}{9\pi(2-\sqrt{3})\cdot 2\cdot 4} = \frac{9}{(2-\sqrt{3})16} = \frac{9(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})16} = \frac{9(2+\sqrt{3})}{16} \Rightarrow$$

$$\Rightarrow \boxed{T\left(0,0,\frac{9(2+\sqrt{3})}{16}\right)}$$