Dodatne naloge za PARCIALNE ODVODE

1. Izračunaj parcialne odvode po vseh spremenljivkah funkcij

   a.) $f(x,y) = x^4 + y^4 - 4x^2y^2$
   b.) $f(x,y) = x\sin(x+y)$
   c.) $f(x,y) = \ln(x+y^2)$
   d.) $f(x,y) = \operatorname{arctg}\frac{x+y}{1-xy}$
   e.) $f(x,y) = x^2 + xy + y^x - x\sin y,\ x \neq 0, y > 0$
   f.) $f(x,y) = \operatorname{arctg}\frac{x}{y}$
   g.) $f(x,y,z) = x\sin(xy^2z^3) + z$
   h.) $f(x,y,z) = (2x+3y-z)^2$

   R: a.) $f_x = 4x^3 - 8xy^2$, $f_y = 4y^3 - 8x^2y$, b.) $f_x = \sin(x+y) + x\cos(x+y)$, $f_y = x\cos(x+y)$, c.) $f_x = \frac{1}{x+y^2}$, $f_y = \frac{2y}{x+y^2}$, d.) $f_x = \frac{1+y^2}{1+(x+y)^2}$, $f_y = \frac{1+x^2}{1+(x+y)^2}$, e.) $f_x = 2x + y + y^x\ln y - \sin y$, $f_y = x + xy^{x-1} - x\cos y$, f.) $f_x = \frac{y}{x^2+y^2}$, $f_y = -\frac{1}{x^2+y^2}$, g.) $f_x = \sin(xy^2z^3) + xy^2z^3\cos(xy^2z^3)$, $f_y = 2x^2yz^3\cos(xy^2z^3)$, $f_z = 3x^2y^2z^2\cos(xy^2z^3) + 1$, h.) $f_x = 4(2x+3y-z)$, $f_y = 6(2x+3y-z)$, $f_z = -2(2x+3y-z)$

2. Zapiši gradient naslednjih funkcij:

   a.) $f(x,y) = y(x^2+1)$
   b.) $f(x,y,z) = x\cos(xy+z)$
   c.) $f(x,y,z) = \sqrt{2x+x^2y^2}$

   R: a.) $\operatorname{grad} f = (2xy, x^2+1)$ b.) $\operatorname{grad} f = (\cos(xy+z) - xy\sin(xy+z), -x^2\sin(xy+z), -x\sin(xy+z))$, c.) $\operatorname{grad} f = \left(\frac{1+xy^2}{\sqrt{2x+x^2y^2}}, \frac{x^2y}{\sqrt{2x+x^2y^2}}\right)$

3. Pokaži, da je $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 1$, če je $f(x,y,z) = x + \frac{x-y}{y-z}$.

4. Pokaži, da $u = \varphi(x)\psi(y)$ zadošča enačbi $u\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} = 0$, kjer sta φ in ψ odvedljivi funkciji.

5. Izračunaj $\frac{\partial^2 z}{\partial x^2}$ in $\frac{\partial^2 z}{\partial x\partial y}$, če je $z = f(x^2+y^2)$ in je funkcija f dvakrat zvezno odvedljiva.

   R: $\frac{\partial^2 z}{\partial x^2} = 4x^2f'(x^2+y^2) + 2f'(x^2+y^2)$, $\frac{\partial^2 z}{\partial x\partial y} = 4xyf''(x^2+y^2)$

6. S pomočjo verižnega pravila izračunaj f_t, kjer je $f(x,y) = x^2+y^2$, $x(r,\varphi) = r\cos\varphi$, $y(r,\varphi) = r\sin\varphi$, $r(t) = t^2+1$ in $\varphi(t) = 2t-2$.

   R: $f_t = 4t(t^2+1)$

7. Izračunaj $\frac{\partial g}{\partial r}$ in $\frac{\partial g}{\partial \alpha}$, če je $g(x,y,z) = xyz^2$, $x(r,\alpha) = r\sin\alpha$, $y(r,\alpha) = r\cos\alpha$ in $z(r) = r^2$.

   R: $\frac{\partial g}{\partial r} = 3r^5\sin 2\alpha$ in $\frac{\partial g}{\partial \alpha} = r^6\cos 2\alpha$

8. Izrazi naslednje izraze s spremenljivkama r in φ (kjer $x = r\cos\varphi$, $y = r\sin\varphi$):

   a.) $u = xz_x + yz_y$
   b.) $u = z_x^2 + z_y^2$

   R: a.) $u = rz_r$, b.) $u = z_r^2 + \frac{z_\varphi^2}{r^2}$